MODELS OF INTEGRATED INVENTORY SYSTEM WITH STOCK-DEPENDENT DEMAND

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Abstract

This study is proposing two integrated inventory models by considering stock-dependent demand. We presented a mathematical model where a vendor procures raw material in single/multiple instalment(s), processes them to make finished products, ships to single/multiple buyer(s) at single/multiple shipment(s) and stores them at the warehouse before presents at the display area with single/multiple transfer(s). The first model operates with a single-vendor and a single-buyer by considering different shipment policies. Two types of shipment policies were considered in the first model: geometric shipment and geometric-then-equal shipment sizes. The optimum joint profits were obtained by using Wolfram Mathematica Version 7. Numerical examples were then presented for discussion. A numerical sensitivity analysis was conducted to investigate the reaction of the optimal total profit to the changes in the parameters value. The second model operates with a single-vendor and multiple buyers by considering equal shipment sizes. The total optimum profit and solution procedures were obtained by using Microsoft Excel Premium Solver Version 12.5 which can solve non-linear optimization problems. This procedure was used to analyze some numerical examples in order to examine the model's sensitivity to parameter changes as well.

Abstrak

Tesis ini mencadangkan dua model inventori bersepadu dengan mempertimbangkan kes permintaan bergantung kepada paras stok. Dalam model ini, penjual mendapatkan bahan mentah dalam satu/berbilang pesanan. Bahan mentah tersebut diproses untuk menghasilkan produk siap, dan dihantar kepada seorang/berbilang pembeli pada satu/berbilang penghantaran. Pembeli akan menyimpan produk di gudang sebelum produk disusun di kawasan paparan dengan satu/berbilang pemindahan. Model pertama merangkumi penjual tunggal dan pembeli tunggal dengan mempertimbangkan dasar penghantaran yang berbeza. Dua jenis dasar penghantaran dicadangkan dalam model pertama: penghantaran geometri dan penghantaran geometri kemudian sama saiznya. Keuntungan bersama optimum bagi model tersebut dicari menggunakan Wolfram Mathematica Versi 7. Contoh berangka diberikan untuk perbincangan. Analisa dilaksanakan untuk mengkaji tindak balas keuntungan optimum dengan perubahan nilai parameter. Model kedua merangakumi penjual tunggal dan berbilang pembeli dengan mempertimbangkan saiz penghantaran sama. Jumlah keuntungan optimum dan prosedur penyelesaian dicari menggunakan perisian Microsoft Excel's Premium Solver Versi 12.5 di dalam Microsoft Excel yang boleh menyelesaikan masalah pengoptimuman bukan linear. Beberapa contoh berangka dan analisa turut dibuat untuk mengkaji tindak balas keuntungan optimum dengan perubahan nilai parameter.

Acknowledgement

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Chapter 1

Introduction

A supply-chain deals with the procurement of raw materials, transforms them into finished products before distributes these products to buyers. It takes a huge amount of capital in the form of plants, equipments and inventories. Figure 1.1 depicts the interactions between raw materials supplier, vendor and buyer. For example, a company procures cocoa beans from the supplier, processes them to make chocolate bars, and ship the products to buyers such as Giant, Carrefour, and Jusco. They will store the products at their own warehouse before being presented to end customers in a display area.

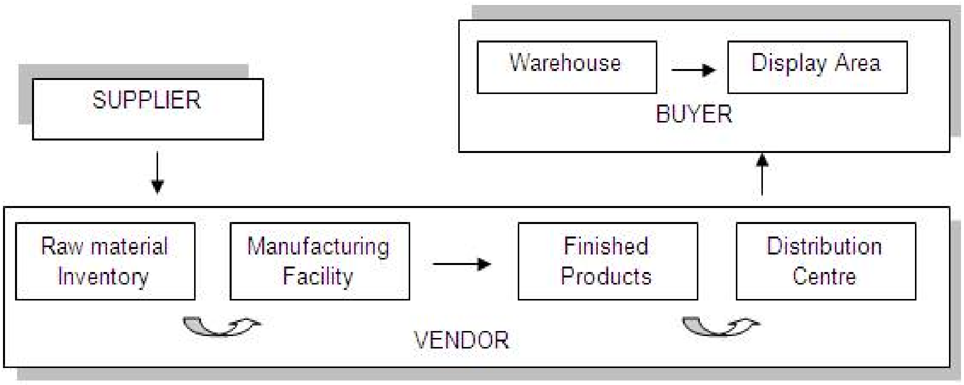

Figure 1.1: Flow diagram of integrated supply chain

In most situations, the demand rate of products is influenced by many factors such as stock level at the showroom, pricing policy, and quality of the products. It is a common scenario where the increase of shelf space for an item encourages more consumers to buy it. This occurs due to its popularity, visibility or variety. Many retail stores stock large piles of goods on their shelf to attract customers and obtain more sale profits. In this research case, the researcher assumes that the demand is dependent on the current stock level.

In order to satisfy customers' demand, it is very important to replenish inventories at the right amount at the right time. The companies must control the inventories effectively in order to avoid having either too much inventories or not enough inventories. Having too many inventories may increase the carrying cost meanwhile insufficient inventories lead to stock-out, loss of customers, and reduction in market share. Therefore, the delicate balance between high and low inventory level, and the cost of inventory need to be minimized.

The common model for inventory control is commonly known as Economical Order Quantity (EOQ). This model has been developed based on a fixed lead time to determine the amount of stock to be ordered. As a simple extension to the EOQ model, the Economic Production Quantity (EPQ) determines the amount of stock to be produced incrementally while products are being produced.

Existing inventory models are focused on a single perspective. For example, the optimal ordering and shipment policies for vendor and buyer are dealt separately. Thus, the lot size of the buyer may not result in an optimal policy for the vendor and vice versa. The huge challenge is to minimize the total costs and enhance customer services throughout the supply chain. In order to be competitive, vendors and buyers need to establish long-term cooperative relationships. This means that an optimal contract quantity and an optimal number of delivery must be set at the beginning of the contract based on their integration of total cost function.

The main purpose of this research is to develop an integrated inventory model for a three-level supply chain which comprises a single raw material supplier, single vendor(manufacturer) and single/multi-buyer(s).

1.1 Objectives

The objective of this research is to study the integrated production-inventory model of a three-level supply chain network. In this system, a vendor procures raw material from a supplier at a single or multiple installments, processes them to make finished products, and ships them to buyer(s) at a single or multiple shipments. The buyer(s) will keep them at their respective warehouse before being presented to the end customers in a display area with a single or multiple transfers. It has been observed in supermarkets that the demand is usually influenced by the amount of stock on the shelves. An increase in shelf space for an item encourages more customers to buy it. Therefore, we assume that the demand is depend on the current stock level at the display area.

The objectives of this research are:

i. To develop a mathematical model for an integrated single-supplier, single-vendor and single-buyer and to find the best solution for a single-supplier, single-vendor and single-buyer by considering unequal shipment sizes policy.

ii. To develop a mathematical model for an integrated single-supplier, single-vendor and multi-buyer and to find the best solution for a single-supplier, single-vendor and multiple-buyer by considering equal shipment sizes policy.

1.2 Thesis overview

This thesis is divided into 5 chapters and will be organized as follow:

Chapter 1: This chapter contains introduction on the fields which are related to the research problems, and objectives of the study.

Chapter 2: This chapter contains review of the relevant literatures and drawbacks of the previous research.

Chapter 3: This chapter involves with development of first model for a single-supplier, single-vendor and single-buyer with unequal shipment sizes policy.

Numerical examples are included.

Chapter 4: This chapter analyzes the second model for a single-supplier, single-vendor and multiple-buyer with equal shipment sizes policy. Numerical examples are included.

Chapter 5: This chapter summarizes the results and present the future studies.

Chapter 2

Literature Review

Goyal (1976) was probably one of the first to introduce the idea of joint optimization for two-level supply chain with single-vendor and single-buyer which minimizes the total relevant costs for both vendor and buyer. He assume that the production rate is infinite and demand is constant over time. The model was generalized by Barnerjee (1986) by relaxing the assumption of infinite production rate, hence introducing the concept of joint economic lot size (JELS) which reduces the total cost for both vendor and purchaser. In this model, a vendor delivers a lot of products which are produced one batch at a time. Goyal (1988) then extended Barnerjee's model (1986) by allowing the production lot to be supplied in n integer number of shipments.

Lu (1995) developed an optimal policy for a single-vendor single-buyer problem whereby the delivery quantities sent to the buyer are similar for every time the stock was replenished. Hill (1997) further determined the geometric shipment policy where the geometric growth factor is a decision variable within a certain range. The vendor delivers a batch quantity of 'Q' in n shipments of sizes equal to $q_1, q_2, ..., q_n$. The successive shipments increased by a factor which value should be in between one and the ratio between the manufacturing rates to the demand rates of the products. He showed that, at a fixed transportation cost per shipment, the total costs were smaller than by using equal sub-batches. Hill (1999) then derived a structure of the globally optimal batching and shipping policies for the single-vendor single-buyer integrated production inventory problem. Another model involving different sizes of shipment was suggested by Goyal and Nebebe (2000). They proposed a simple geometric-then-equal policy where the constant factor of the first shipment size, q followed by the size of the remaining equal shipments, each equal to the production rate divided by the demand rate, x and multiply by q. They showed that their method often achieve better result compared to Goyal (1995), Lu (1995), and Hill (1997). Goyal (2000) considered another policy where the following shipment sizes would be determined by first shipment size.

In any production, when a vendor (manufacturer) uses raw materials, the ordering quantity of raw materials are dependent on the batch production quantity of the finished products and thus isolates the economic ordering problem of raw materials from economic batch quantity which is undesirable. It is preferable to determine the optimum batch quantities or production cycle time together with its raw material ordering quantities. Some authors have dealt with this kind of problems such as Golhar and Sarker (1992), Jamal and Sarker (1993), and Sarker and Parija (1994). They developed an integrated model to find an optimal or near-optimal solution for manufacturing batch size and ordering policy for procurement of raw materials to minimize the total cost by considering equal shipments of the finished products, or by delivering at fixed interval to the buyer. Sarker and Parija (1996) considered an optimal multiple ordering procurement policy for raw material for a single stage manufacturing batch. Hill and Omar (2006) summarized the prior research on the single vendor single-buyer integrated production inventory model and relaxes the consignment case by considering the batch dimensions on a replenishment cycle.

For a three level supply chain, Banerjee and Kim (1995) presented their model from an integrated standpoint of the buyer, the manufacturer, and the raw materials supplier in a Just-In-Time (JIT) environment. Munson and Rosenblatt (2001) showed that the benefits of using quantity discount from coordinated lot sizing among three level supply chains is to decrease cost. They derived their solution approach by exploring the optimality structure of the optimal cost function with respect to the ordering quantity from the vendor. Jaber et al. (2006) extended the work of Munson and Rosenblatt (2001) by considering ordered quantity and price as decision variables.

A common assumption made by researches above is that demand is exogenous. However, it has been recognized in the marketing literature that demand for certain items for example in a supermarket is influenced by the amount of stock displayed in the shelves. As pointed out by Levin et al. (1972) and Silver and Peterson (1985) that the sales quantity of some company is proportional to its displayed product, such as supermarket. Gupta and Vrat (1986) were among the first to incorporate this observation into an inventory model where the demand rate is a function of initial stock level. Baker and Urban (1988) discussed an inventory model with an inventory level dependent demand. Mandal and Phaujdar (1989) corrected the flaws in Gupta and Vrat (1986) model by using profit maximization instead of cost minimization as the objective function. Datta and Pal (1990) modified the model of Baker and Urban (1988) by assuming that the stock dependent demand rate was down to a given level of inventory, beyond which the demand rate becomes constant. Goh (1994) relaxed the assumption of a constant holding cost in Baker and Urban (1988) and Dye and Ouyang (2005) extended the classical economic order quantity model to allow for the demand rate depended on both selling price and displayed stock level. They allowed shortages and general partial backlogging. Goyal and Chang (2009) developed an ordering-transfer inventory model by assuming the amount of display space is limited and the demand rate depends on the display stock level. They derived their solution to determine optimal order quantity and the number of transfers per order from the warehouse to the display area. Recently, Sajadieh et al., (2010) proposed an integrated inventory model for a single-vendor single-buyer problem with limited display area and the demand rate depends on the displayed stock level. They assumed that the vendor delivers the finished products in multiple shipments of equal lot batch size to the buyer. They determine the replenishment policy in term of three variables of q(lot sizes), n(number of shipments), and m(number of transfers). Glock (2012a) gave a comprehensive review of joint economic lot size problems.

2.1 Single-vendor multiple-buyer problem

In coping with today's complex supply chain, it is important to understand extended systems comprising multiple buyers. Compared with the single-vendor-single buyer system, the one-vendor multiple-buyer problem has been known to be non-trivial.

A single-vendor multiple-buyer problem was addressed by Affisco et al. (1988, 1991, and 1993). In these studies, they addressed the objective of replacing the production setup cost and the retailer's ordering cost . They showed that substantial improvement could be achieved under this model through the independent cost optimization technique. Therefore, in a cooperative environment, an integrated inventory approach is suggested over independent cost optimization. Joglekar and Tharthare (1990) presented an individually responsible and rational decision (IRRD) approach to the economic lot sizes for one vendor and multiple identical buyers. In IRRD, they refined Joint Economic Lot Sizing (JELS) by breaking setup cost into vendor's order processing and handling cost per production run setup cost. They showed that their approach under IRRD reduced system costs more than the system cost under the JELS approach.

Banerjee and Burton (1994) developed an integrated production inventory model for a single-vendor and multiple buyers under deterministic conditions. To avoid shortages at the vendor's side, they considered a delivery cycle time, common to all buyers, and a supplier's manufacturing cycle which was assumed to be an integer multiple of the delivery cycle time. They found that the optimal solution was more superior if the echelons agree to collaborate in implementing such a system compared to independent optimization. Lu (1995) proposed his model of a single-vendor or multiple-buyer scenario. He considered shipment during production, which were antithetical with Goyal's(1995) assumption. Lu's model is more practical than previous researches as he included historical purchasing information and highest expected acceptable cost in determining the minimum value of the total setup cost and inventory cost. Viswanathan and Piplani (2001) studied a single-vendor multiple buyers supply chain for a single item to study the benefit of coordinating the supply chain through common replenishment epoch (CRE). They assumed that the vendor holds no inventory and orders the products through a supplier whenever an order is received from buyers.

Wee and Yang (2002) evaluated a single-vendor multiple buyers production-inventory policy for a deteriorating item. Siajadi et al. (2006) proposed a single-vendor multiple buyers case where the shipments were equal. They assumed that the production cycle time and the order cycle time for the buyers were equal and the supplies are shipped in sequence. The shipment size from each buyer might differ based on their demand.

Very few are those researches that assumed more than one actor at each level of a three-level supply chain. Khouja (2003) developed a supply chain where each stage of multiple firms and a firm can ship two or more buyers. Wee and Yang (2004) developed a heuristic solution model of the integrated system of a single-supplier, multiple vendors and multiple buyers by revising Goyal's model (1988). Jaber and Goyal (2008) considered multiple suppliers, a manufacturer, and multiple buyers. They assumed that a supplier may supply one or more items to the vendor before manufacture the items into a single product and shipped simultaneously to buyers. They also considered a case where the vendor could act as a supplier of some of the items, which a fourth level to the chain.

2.2 Drawbacks of previous research

After reviewing the literature, it is realized that both researchers as well as practitioners have shown interest toward the integrated inventory supply chain system. The review points out that there are some flaws in the earlier researches. The differences in shipment sizes attract the attention of few researchers in developing the inventory model, but they only considered the deterministic demand. The aforementioned literatures have studied the inventory model strictly with two echelons i.e., a single-vendor and a single-buyer problem. Higher order stages and echelons need to be explored.

Based on all researches and shortcomings mentioned above, an integrated inventory model with stock-dependent demand with unequal sized of shipment is performed as the prior development in the first model instead of equal sized of shipment. Next, the previous paper proposed by Sajadieh.et.al (2010) is extended into three-level supply chain and assume multiple buyers problem.

Chapter 3

Single-Supplier, Single-Vendor and Single-Buyer

In this chapter, we revisit a single-vendor and single-buyer with stock-dependent demand. We will extend Sajadieh et.al (2010) by adding another echelon, a single-supplier and considering unequal shipment sizes. The objective is to maximize the total profit of the supply chain. The current researcher obtained optimal solution in terms of the optimal value of λ, transfer lot sizes (q_1), number of installments (n_r), number of shipments (n_v), and number of transfers (n_b) which gives the maximum total profit, TP^* by using Wolfram Mathematica Version 7.

3.1 Assumptions

1. The demand rate in period-i, $D_i(t)$ is assumed to be in the form as in Baker and Urban (1988), i.e. $D_i(t) = \alpha[I_i(t)]^\beta$, where $I_i(t)$ is the stock level in display of item in ith shipment, $\alpha > 0$ and $0 < \beta < 1$ are the scale and the shape parameter, respectively. The shape parameter β, reflects the elasticity of the demand rate with respect to the stock level on display.

2. Shortages at the buyer warehouse and display area are not allowed.

3. Time horizon is infinite.

4. There is a limited capacity, C_d of the display area, i.e $I_i(t) \leq C_d$. This limitation could be interpreted as a given shelf space, allocated to the product.

5. The vendor has a finite production rate P which is greater than the maximum possible demand rate, i.e $P > \alpha C_d^{\beta}$.

6. For simplicity, only one type of raw material is considered as required to produce one unit of a finished item.

3.2 Notations

The following notational scheme is adopted:

1. A_v Setup cost per production for vendor,v.

2. A_b Fixed shipment cost for buyer,b.

3. A_r Fixed installment cost for raw material,r.

4. S Fixed transferring cost for buyer from the warehouse to the display area.

5. c The net unit purchasing price (charged by the vendor to the buyer).

6. σ The net unit selling price (charged by the buyer to the consumer).

7. h_r The raw material holding cost per unit time.

8. h_v The inventory holding cost per unit time at the vendor.

9. h_w The inventory holding cost per unit time at the buyer's warehouse,w where $h_w > h_v$.

10. h_d The inventory holding cost per unit time at the buyer's display area,d where $h_d > h_w$.

11. n_r The number of installment.

12. n_v The number of shipment.

13. n_b The number of transfer.

14. Q_i The shipment lot size from vendor to buyer warehouse where $i = 1, 2, ..., n_v$.

15. q_i The transfer lot size from warehouse to display area where $i = 1, 2, ..., n_v$.

16. λ Geometric growth factor.

3.3 General formulation

At the beginning of the cycle, a single vendor will procure raw material from a single supplier at n_r multiple installments, processes them to make finished items and ships them to a single buyer at n_v multiple shipments. The buyer will store them at the warehouse before being presented to the end customers at display area with n_b multiple transfers. The graphical representation of the system is presented in Figure 3.1.

We let $I_i(t)$ be the display area inventory level at time t, and we have

$$\frac{dI_i(t)}{dt} = -\alpha[I_i(t)]^{\beta}, \quad 0 \leq t \leq T_{d_i}, \quad i = 1, 2, ..., n_v. \tag{3.1}$$

where $i = 1, 2, 3, ..., n_v$ and T_{d_i} is the cycle time at the display area with $I_i(0) = \frac{Q_i}{n_b}$ and $I_i(T_{d_i}) = 0$.

Therefore, the (on-hand) inventory at time t can be obtained by solving (3.1) which we get

$$I_i(t) = \Big[- \alpha(1-\beta)t + (\frac{Q_i}{n_b})^{1-\beta} \Big]^{\frac{1}{1-\beta}}.$$

Substituting $I_i(T_{d_i}) = 0$ into the above expression, we get the formulation for T_{d_i} depend on the transfer quantity q_i as $T_{d_i} = \frac{1}{\alpha(1-\beta)} \left(\frac{Q_i}{n_b} \right)^{1-\beta}$, where $Q_i = n_b q_i$.

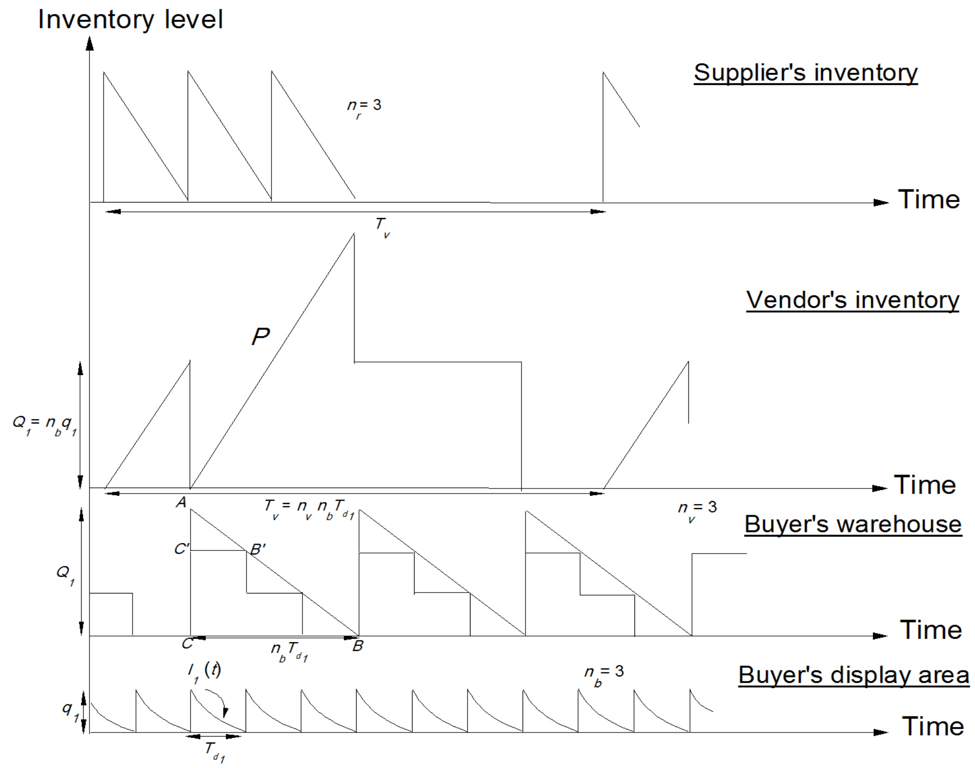

Figure 3.1: A graphical representation of integrated supply chain with equal shipment sizes policy.

3.3.1 Average total profit formulation

The total profit is a combination of total cost and total revenue of the buyer and vendor. This section deals with the derivation of the general formulation for the average total cost that will be used for the entire shipment policies.

3.3.1.1 Buyer's average total cost formulation

At first shipment, the buyer receives Q_1 amount of an item, keeps the items inside the warehouse and transfers n_b times q_1 of amount until the items at the warehouse reach zero. The time taken to consume Q_1 is $n_b T_{d_1}$. The buyer's inventory at the warehouse can be obtained from the summation of the shipments where for each shipment, the inventory is denoted with the area of triangle ABC subtracts with the area of triangle $AB'C'$ times n_b. Thus, the buyer's inventory at the warehouse for the first shipment is $[(n_b - 1)Q_1 T_{d_1}]$. This continues until entire shipment is supplied. The current researcher assumed that there are n_v shipment in a lot, thus the total average inventory at the warehouse per cycle is given by

$$\begin{aligned} I_{bw} &= \frac{1}{2T_v}\Big[(n_b-1)Q_1T_{d_1} + (n_b-1)Q_2T_{d_2} + ... + (n_b-1)Q_{n_v}T_{d_{n_v}}\Big] \\ &= \frac{(n_b-1)}{2T_v}\Big(Q_1T_{d_1} + Q_2T_{d_2} + ... + Q_{n_v}T_{d_{n_v}}\Big). \end{aligned} \tag{3.2}$$

where T_v is the total cycle time of the supply chain and from Figure 3.1, we have

$$T_v = n_b \sum_{i=1}^{n_v} T_{d_i} = \frac{n_b}{\alpha(1-\beta)} \sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}.$$

By solving and simplify Equation(3.2), the average inventory at the warehouse can be expressed as follows:

$$I_{bw} = \frac{(n_b-1)\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{2-\beta}}{2\sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{1-\beta}} \tag{3.3}$$

The total holding cost at the warehouse is given by

$$\begin{aligned} HC_{bw} &= h_w I_{bw} \\ &= \frac{h_w(n_b-1)\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{2-\beta}}{2\sum_{i=1}^{n_v}(\frac{Q_i}{n_b})^{1-\beta}}. \end{aligned} \tag{3.4}$$

Now, the expression of the average inventory at the display area needs to be defined. From Figure 3.1, it can be observed that the average inventory holding of the items during the $(i+1)^{th}$ batch is given by $n_b \int_0^{T_{d_{i+1}}} I_{i+1}(t)dt$. Then, it follows the average inventory at the display area, I_{bd} is given by

$$\begin{aligned} I_{bd} &= \frac{n_b}{T_v}\sum_{i=0}^{n_v-1}\int_0^{T_{d_{i+1}}} I_{i+1}(t)dt \\ &= \frac{n_b}{T_v}\Big[\int_0^{T_{d_1}} I_1(t)dt + \int_0^{T_{d_2}} I_2(t)dt + ... + \int_0^{T_{d_{n_v}}} I_{n_v}(t)dt\Big]. \end{aligned} \tag{3.5}$$

Thus, by solving and simplifying the expression above, the equation of the average inventory at the display area can be rewritten as

$$I_{bd} = \frac{(1-\beta)\sum_{i=1}^{n_v} q_i \left(\frac{Q_i}{n_b}\right)^{2-\beta}}{(2-\beta)\sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}}. \tag{3.6}$$

The total holding cost at the display area is given by

$$\begin{aligned} HC_{bd} &= h_d I_{b_d} \\ &= \frac{h_d(1-\beta)\sum_{i=1}^{n_v} q_i \left(\frac{Q_i}{n_b}\right)^{2-\beta}}{(2-\beta)\sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} \end{aligned} \tag{3.7}$$

Finally, the total relevant cost at the buyer,

$$TRC_b = \frac{1}{T_v}(n_v A_b + n_v n_b S) + HC_{bw} + HC_{bd}. \tag{3.8}$$

where the first term represents the average shipment cost at the warehouse and average transfer cost at the display area.

3.3.1.2 Vendor's average total cost formulation

At the end of a production run, all units of raw material will be fully consumed. Let ψ be the total production quantity, and from Figure 3.1, the maximum inventory of raw material for each installment is ψ/n_r and lasting for the period of $\psi/(n_r P)$ units time. It follows that the average inventory of raw material, I_r is

$$I_r = \frac{1}{T_v}(\frac{\psi}{n_r})(\frac{\psi}{2n_r P}n_r) = \frac{\psi^2}{n_r P T_v} \tag{3.9}$$

where $\psi = \sum_{i=1}^{n_v} Q_i$.

Thus, the holding cost of raw material is

$$HC_r = h_r I_r$$

$$= h_r \frac{\psi^2}{n_r P T_v}. \tag{3.10}$$

Now, by following the same method as in Hill and Omar (2006), the average finished item at the vendor, I_v is $\frac{\psi}{2} - \frac{\psi^2}{2T_v P} + \frac{\psi Q_1}{2T_v P} - \frac{n_b}{2T_v} \sum_{i=1} n_v Q_i T_{d_i}$ where the first three terms represent the average system stock and the final term represent the average stock with the shipment size of Q_i for $i =, 1, 2, ..., n_v$. Thus, by substitution, we get

$$\begin{aligned} I_v &= \frac{\sum_{i=1}^{n_v} Q_i}{2} - \frac{\alpha(1-\beta)\sum_{i=1}^{n_v} Q_i^2}{2n_b P \sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} + \frac{\alpha(1-\beta) Q_1 \sum_{i=1}^{n_v} Q_i}{n_b P \sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} \\ &- \frac{\sum_{i=1}^{n_v} Q_i \left(\frac{Q_i}{n_b}\right)^{1-\beta}}{2\sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}}. \end{aligned} \tag{3.11}$$

It follows that the total holding cost at the vendor is

$$\begin{aligned} HC_v &= h_v I_v \\ &= h_v \Big[\frac{\sum_{i=1}^{n_v} Q_i}{2} - \frac{\alpha(1-\beta)\sum_{i=1}^{n_v} Q_i^2}{2n_b P \sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} + \frac{\alpha(1-\beta) Q_1 \sum_{i=1}^{n_v} Q_i}{n_b P \sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} \\ &- \frac{\sum_{i=1}^{n_v} Q_i \left(\frac{Q_i}{n_b}\right)^{1-\beta}}{2\sum_{i=1}^{n_v} \left(\frac{Q_i}{n_b}\right)^{1-\beta}} \Big]. \end{aligned} \tag{3.12}$$

Finally, the total relevant cost at the vendor,

$$TRC_v = \frac{1}{T_v}(A_v + n_r A_r) + HC_r + HC_v \tag{3.13}$$

where the first term represents average setup cost and average installment cost at the vendor.

3.3.1.3 Vendor-buyer sales revenue

In this supply chain system, the vendor produces $\sum_{i=1}^{n_v} Q_i$ items and sells to the buyer at price c per unit item. Thus, the vendor's sale revenue per unit time is $\frac{c\sum_{i=1}^{n_v} Q_i}{T_v}$. For each order with a quantity of Q_i, the buyer is charged cQ_i from the vendor, and receives the amount γQ_i from the customer. Therefore the buyer's total sale revenue per unit time is $\frac{(\gamma-c)\sum_{i=1}^{n_v} Q_i}{T_v}$. Once the vendor and buyer have established long-term strategic partnership and contracted to commit the relationship, they will jointly determine the best policies for the whole supply chain system. Hence, the total joint expected sales revenue for vendor and buyer is

$$\begin{aligned} TJR &= \frac{c\sum_{i=1}^{n_v} Q_i}{T_v} + \frac{(\gamma - c)\sum_{i=1}^{n_v} Q_i}{T_v} \\ &= \frac{\gamma\alpha(1-\beta)\sum_{i=1}^{n_v} Q_i}{n_v \sum_{i=1}^{n_v}\left(\frac{Q_i}{n_b}\right)^{1-\beta}} \end{aligned} \tag{3.14}$$

Finally, the total joint profit per unit time, TP for the integrated model is

$$TP = TJR - TRC_v - TRC_b. \tag{3.15}$$

The problem is to maximize TP by seeking the optimal control variables value with different shipment policies. All policies are considered below.

3.3.2 Policy 1 : Geometric shipment

In this model, we assume that the shipment sizes increase by a factor of λ. For the first case, the value of λ is fixed to P/α (geometric policy with fixed λ, GF policy). For the second case, λ is a variable within 1 and P/α (geometric policy with variable λ, GV policy). The inventory level time plot for a geometric shipment sizes with $n_b = 4$, $n_v = 2$, and $n_r = 2$ as depicted in the Figure 3.2.

The top part of the figure shows the inventory level of raw materials delivered in n_r equal lot installment during production uptime. The bottom part of the figure shows the inventory level of the buyer at display area which were transferred in n_b equal batch size q_i where $q_i = Q_i/n_b$. The inventory level at the vendor and warehouse with shipment size Q_i are shown in the middle part of the figure.

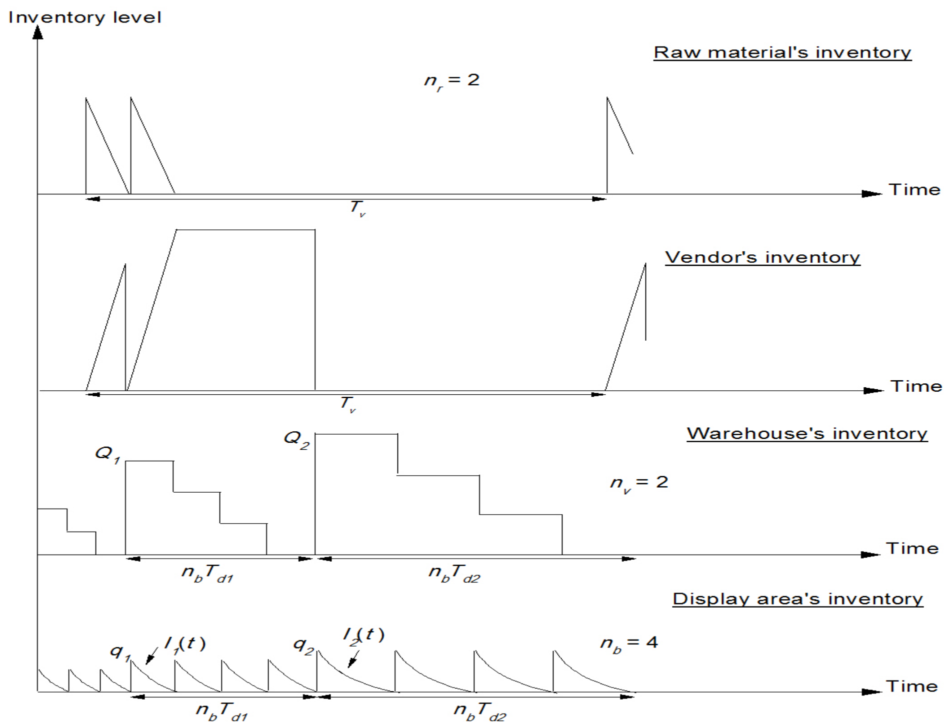

Figure 3.2: A graphical representation of integrated supply chain with geometric shipment sizes policy.

As soon as production for the first shipment Q_1 is complete, items are shipped to the buyer, and inventory at the vendor drops to zero. Then, the items are transferred from the warehouse to the display area in sized q_1 until the inventory level in the warehouse is depleted to zero. The production is continues and the size for the next shipment is (λQ_1). The shipment size increases with a multiplication factor of λ. This is repeated until the entire shipment is produced.

Thus, the general shipment sizes for this policy is $Q_i = \lambda^{i-1}Q_1$, where $i = 2, 3, ..., n_v$. It follows that $q_i = \lambda^{i-1}q_1$ where $q_i = Q_i/n_b$. For equal shipment size we have $\lambda = 1$.

Substitute into equations (3.4), (3.7) and (3.8), then we have

$$HC_{bd} = h_d \frac{q_1(1-\beta)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(2-\beta)(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})}, \tag{3.16}$$

$$HC_{bw} = h_w \frac{q_1(n_b - 1)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{2(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})}, \tag{3.17}$$

and

$$\begin{aligned} TRC_b &= \frac{1}{T_v}(n_v A_b + n_v n_b S) + h_d \frac{q_1(1-\beta)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(2-\beta)(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})} \\ &+ h_w \frac{q_1(n_b - 1)(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{2(\lambda^\beta - \lambda^2)(\lambda^{n_v} - \lambda^{n_v\beta})}. \end{aligned} \tag{3.18}$$

Similarly, from equations (3.10), (3.12) and (3.13), we have

$$HC_r = h_r \frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta} \alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^\beta - \lambda)}{2n_r P(-1+\lambda)^2(\lambda^{n_v\beta} - \lambda^{n_v})}, \tag{3.19}$$

$$\begin{aligned} HC_v &= h_v \frac{n_b q_1}{2}\Big[- \frac{(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(-\lambda^2 + \lambda^\beta)(\lambda^{n_v} - \lambda^{n_v\beta})} \\ &+ \frac{-1+\lambda^{n_v}}{-1+\lambda}\Big(1 \\ &- \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^\beta)}{P(\lambda^{n_v} - \lambda^{n_v\beta})}(\frac{-1+\lambda^{n_v}}{-1+\lambda} - 2)\Big)\Big] \end{aligned} \tag{3.20}$$

and

$$\begin{aligned} TRC_v &= \frac{1}{T_v}(n_r A_r + A_v) + h_r \frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta} \alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^\beta - \lambda)}{2n_r P(-1+\lambda)^2(\lambda^{n_v\beta} - \lambda^{n_v})} \\ &+ h_v \frac{n_b q_1}{2}\Big[- \frac{(\lambda^\beta - \lambda)(\lambda^{2n_v} - \lambda^{n_v\beta})}{(-\lambda^2 + \lambda^\beta)(\lambda^{n_v} - \lambda^{n_v\beta})} + \frac{-1+\lambda^{n_v}}{-1+\lambda} \\ &\Big(1 - \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^\beta)}{P(\lambda^{n_v} - \lambda^{n_v\beta})}(\frac{-1+\lambda^{n_v}}{-1+\lambda} - 2)\Big)\Big]. \end{aligned} \tag{3.21}$$

For this policy, we have $T_v = \frac{n_b q_1 \lambda^\beta q_1^{-\beta}(1-\lambda^{n_v(1-\beta)})}{\alpha(\lambda^\beta - \lambda)(1-\beta)}$. From equations (3.14), we have

$$TJR = \gamma \frac{q_1^\beta \alpha(1-\beta)\lambda^{\beta(n_v-1)}(1-\lambda^{n_v})(\lambda^\beta - \lambda)}{n_b(\lambda - 1)(\lambda^{n_v} - \lambda^{n_v\beta})} \tag{3.22}$$

By substituting equations (3.22), (3.21), and (3.18) into equation (3.15), we get

$$
\begin{aligned}
TP &= \gamma \frac{q_1^{\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(1-\lambda^{n_v})(\lambda^{\beta}-\lambda)}{n_b(\lambda-1)(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&- \frac{n_v q_1^{-1+\beta}\alpha(\beta-1)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{\lambda^{n_v}-\lambda^{n_v\beta}}\Big[\frac{A_v+n_rA_r}{n_bn_v}+\frac{A_b}{n_b}+S\Big] \\
&- h_r\frac{n_b(-1+\lambda^{n_v})^2 q_1^{1+\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{2n_rP(-1+\lambda)^2(\lambda^{n_v\beta}-\lambda^{n_v})} \\
&- h_d\frac{q_1(1-\beta)(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{(2-\beta)(\lambda^{\beta}-\lambda^2)(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&- h_w\frac{q_1(n_b-1)(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{2(\lambda^{\beta}-\lambda^2)(\lambda^{n_v}-\lambda^{n_v\beta})} \\
&- h_v\frac{n_bq_1}{2}\Big[-\frac{(\lambda^{\beta}-\lambda)(\lambda^{2n_v}-\lambda^{n_v\beta})}{(-\lambda^2+\lambda^{\beta})(\lambda^{n_v}-\lambda^{n_v\beta})}+\frac{-1+\lambda^{n_v}}{-1+\lambda}\Big(1 \\
&- \frac{q_1^{\beta}\alpha(1-\beta)\lambda^{\beta(n_v-1)}(-\lambda+\lambda^{\beta})}{P(\lambda^{n_v}-\lambda^{n_v\beta})}\big(\frac{-1+\lambda^{n_v}}{-1+\lambda}-2\big)\Big)\Big]. \qquad (3.23)
\end{aligned}
$$

where $(1/T_v)(n_vA_b+n_vn_bS+n_rA_r+A_v) = \frac{n_vq_1^{-1+\beta}\alpha(\beta-1)\lambda^{\beta(n_v-1)}(\lambda^{\beta}-\lambda)}{\lambda^{n_v}-\lambda^{n_v\beta}}\Big[\frac{A_v+n_rA_r}{n_bn_v}+\frac{A_b}{n_b}+S\Big]$. TP is a function in n_r, n_v, n_b, q_1, and λ. If $\lambda=1$, we will discover an equal shipment sizes policy (ES policy).

3.3.3 Policy 2: Geometric then equal shipment

In this policy (GE policy), we assume that after the first shipment, Q_1, the remaining (n_v-1) shipments, each equal to $\lambda = P/\alpha$ multiply by Q_1. The inventory level time plot for a geometric then equal shipment sizes with $n_b=3$, $n_v=3$, and $n_r=4$ depicted in the Figure 3.3. The inventory level at the vendor and warehouse with shipment size Q_i are shown in the middle part of the figure. The bottom part of the figure shows the inventory level of the buyer at display area.

As soon as production for the first shipment Q_1 is complete, items are shipped to the buyer, and inventory at the vendor drops to zero. Then, the items are transferred from the warehouse to the display area in sized q_1 until the inventory level in the warehouse is depleted to zero. The production continues with the remaining $(n_v - 1)$ shipments are equal to (λQ_1).

It follows that $q_i = \lambda q_1$ where $q_i = Q_i/n_b$ for $i = 2, 3, ..., (n_v - 1)$. Thus, the total shipment for a complete cycle are equal to $\sum_{i=1}^{n_v} Q_i = Q_1\Big(1 + (n_v - 1)\lambda\Big)$. If we put value of $\lambda = 1$, we will discover equation for equal shipment sizes policy (ES policy).

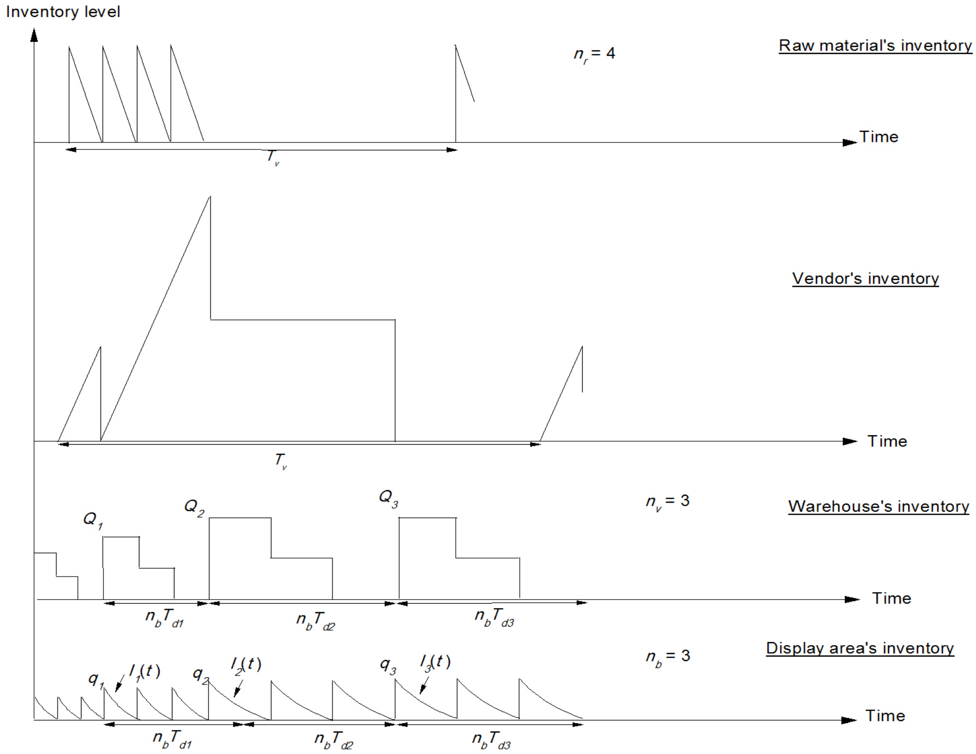

Figure 3.3: A graphical representation of integrated supply chain with geometric then equal shipment sizes policy.

Substitute into equations (3.4), (3.7) and (3.8), then we have

$$HC_{bd} = h_d \frac{q_1(1-\beta)[(n_v - 1)\lambda^{2-\beta} + 1]}{(2-\beta)[(n_v - 1)\lambda^{1-\beta} + 1]} \tag{3.24}$$

$$HC_{bw} = h_w \frac{(n_b - 1)q_1[(n_v - 1)\lambda^{2-\beta} + 1]}{2[(n_v - 1)\lambda^{1-\beta} + 1]} \tag{3.25}$$

and

$$
\begin{aligned}
TRC_b &= \frac{1}{T_v}(n_v A_b + n_v n_b S) + h_d \frac{q_1(1-\beta)[(n_v-1)\lambda^{2-\beta}+1]}{(2-\beta)[(n_v-1)\lambda^{1-\beta}+1]} \\
&+ h_w \frac{(n_b-1)q_1[(n_v-1)\lambda^{2-\beta}+1]}{2[(n_v-1)\lambda^{1-\beta}+1]}. \qquad (3.26)
\end{aligned}
$$

Similarly, from equations (3.10), (3.12) and (3.13), we have

$$
HC_r = h_r \frac{q_1^{1+\beta}\alpha(1-\beta)n_b[(n_v-1)\lambda+1]^2}{2n_r P(n_v-1)\lambda^{1-\beta}+1}, \qquad (3.27)
$$

$$
\begin{aligned}
HC_v &= h_v \frac{n_b q_1}{2}\Big[[(n_v-1)\lambda+1]\Big(1-\frac{q_1^{\beta}\alpha(1-\beta)[(n_v-1)\lambda-1]}{P[(n_v-1)\lambda^{1-\beta}+1]}\Big) \\
&- \frac{(n_v-1)\lambda^{2-\beta}+1}{(n_v-1)\lambda^{1-\beta}+1}\Big] \qquad (3.28)
\end{aligned}
$$

and

$$
\begin{aligned}
TRC_v &= \frac{1}{T_v}(n_r A_r + A_v) + h_r \frac{q_1^{1+\beta}\alpha(1-\beta)n_b[(n_v-1)\lambda+1]^2}{2n_r P(n_v-1)\lambda^{1-\beta}+1} \\
&+ h_v \frac{n_b q_1}{2}\Big[[(n_v-1)\lambda+1]\Big(1-\frac{q_1^{\beta}\alpha(1-\beta)[(n_v-1)\lambda-1]}{P[(n_v-1)\lambda^{1-\beta}+1]}\Big) \\
&- \frac{(n_v-1)\lambda^{2-\beta}+1}{(n_v-1)\lambda^{1-\beta}+1}\Big]. \qquad (3.29)
\end{aligned}
$$

For this policy, we have $T_v = \frac{n_b q_1^{1-\beta}[(n_v-1)\lambda^{1-\beta}+1]}{\alpha(1-\beta)}$.

From equations (3.14), we have

$$
TJR = \frac{\gamma q^{\beta}\alpha(1-\beta)\lambda^{\beta}[(n_v-1)\lambda+1]}{(n_v-1)\lambda+\lambda^{\beta}}. \qquad (3.30)
$$

By substituting equations (3.30), (3.29), and (3.26) into equation (3.15), we get

$$
\begin{aligned}
TP &= \frac{\gamma q^{\beta}\alpha(1-\beta)\lambda^{\beta}[(n_v-1)\lambda+1]}{(n_v-1)\lambda+\lambda^{\beta}} \\
&- \frac{n_v q_1^{-1+\beta}\alpha(1-\beta)}{(n_v-1)\lambda^{1-\beta}+1}\Big(\frac{A_b}{n_b}+S+\frac{A_v+n_rA_r}{n_vn_b}\Big) \\
&- h_d\frac{q_1(1-\beta)[(n_v-1)\lambda^{2-\beta}+1]}{(2-\beta)[(n_v-1)\lambda^{1-\beta}+1]} - h_w\frac{(n_b-1)q_1[(n_v-1)\lambda^{2-\beta}+1]}{2[(n_v-1)\lambda^{1-\beta}+1]} \\
&- h_r\frac{q_1^{1+\beta}\alpha(1-\beta)n_b[(n_v-1)\lambda+1]^2}{2n_rP(n_v-1)\lambda^{1-\beta}+1} - h_v\frac{n_bq_1}{2}\Big[[(n_v-1)\lambda+1] \\
&\Big(1-\frac{q_1^{\beta}\alpha(1-\beta)[(n_v-1)\lambda-1]}{P[(n_v-1)\lambda^{1-\beta}+1]}\Big) - \frac{(n_v-1)\lambda^{2-\beta}+1}{(n_v-1)\lambda^{1-\beta}+1}\Big]. \qquad (3.31)
\end{aligned}
$$

where TP is a function in n_r, n_v, n_b, and q_1.

3.4 Numerical examples

A number of examples are made to test the solution of the models. We employ Wolfram Mathematica Version 7 by using built-in Mathematica. We use built-in Mathematica symbol of $NMaximize[\{f, cons\}, \{x, y, ...\}]$ to maximize f numerically subject to the constraints *cons*. The constraints used in this model are:

i. $q_1 \geq 1$ && $q_1 \leq C_d$.

ii. $n_v \geq 1$ && $n_b \geq 1$ && $n_r \geq 1$.

iii. $\{n_v, n_b, n_r\} \in Integers$.

iv. $\lambda \geq 0.99999$ && $\lambda \leq P/\alpha$. (Valid for GV shipment)

By putting the value of λ approaching to one (to avoid division by zero), and $h_r = A_r = 0$ together with the same parameters values used in Sajadieh et al., (2010) into Equation (3.23) and (3.31), similar numerical results were identified as in their paper.

Example 3.4.1: The concavity of TP^* against variables n_b, n_v, n_r, q_1, and λ

are demonstrated numerically with certain parameters values. The results are presented graphically in Figures 3.4 to 3.8. Assuming to be continuous, by taking the second partial derivative of Equation (3.23) and (3.31), it can be shown easily that TP for both policies are concave in n_b and n_r. However, the concavity behaviour cannot be shown analytically for other decision variables because the functional form is too cumbersome.

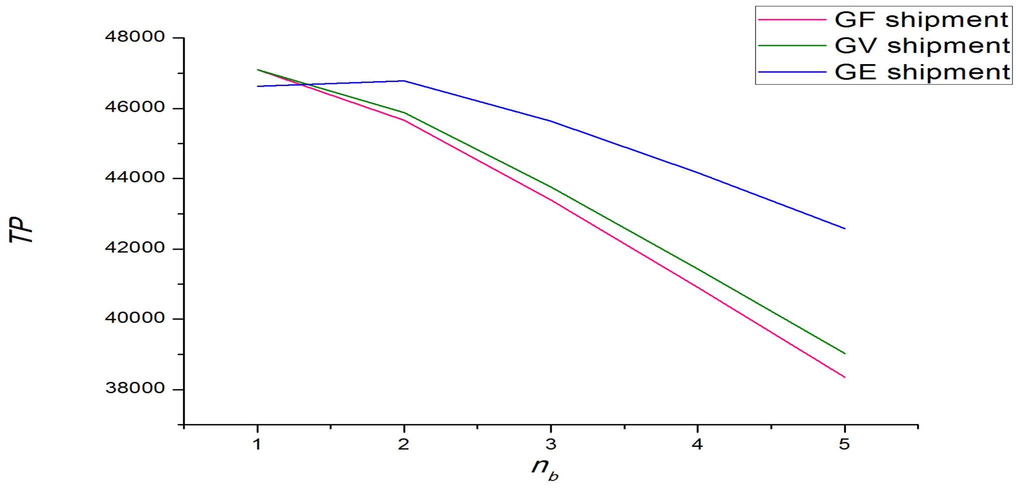

Figure 3.4: Plot of TP^* against n_b when $n_v = 3$, $n_r = 2$, $q_1 = 71.988$, and $\lambda = 2.2675/\frac{P}{\alpha}$.

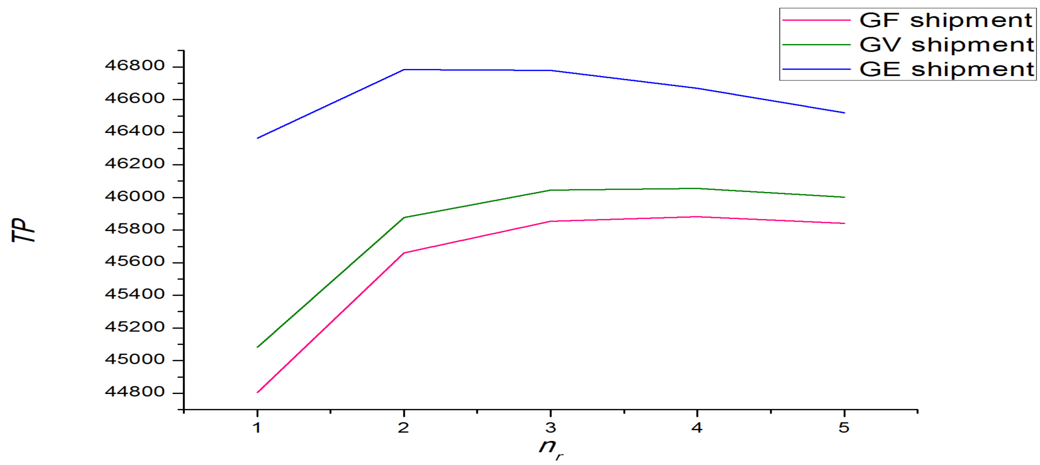

Figure 3.5: Plot of TP^* against n_r when $n_v = 3$, $n_b = 2$, $q_1 = 71.988$, and $\lambda = 2.2675/\frac{P}{\alpha}$.

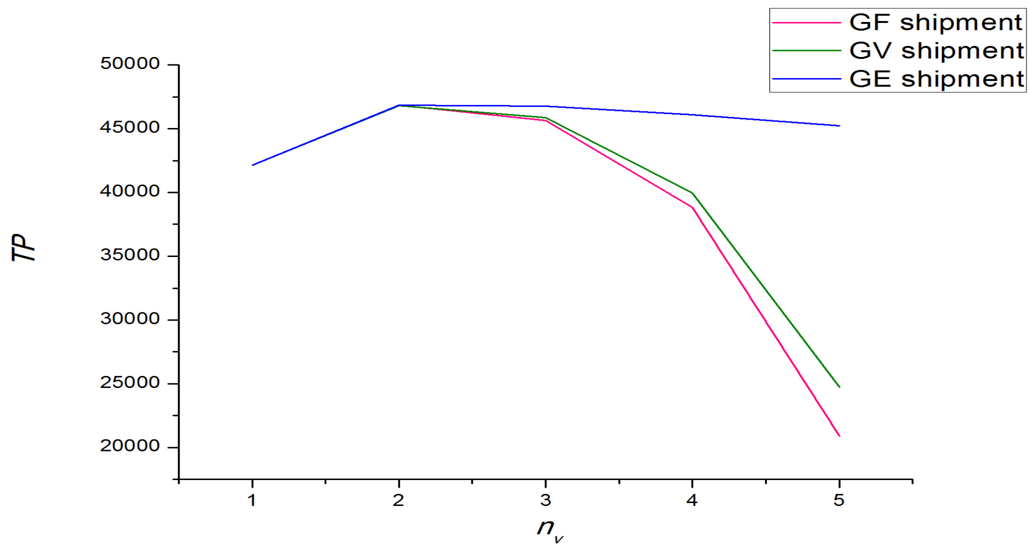

Figure 3.6: Plot of TP^* against n_v when $n_b = 2$, $n_r = 2$, $q_1 = 71.988$, and $\lambda = 2.2675/\frac{P}{\alpha}$.

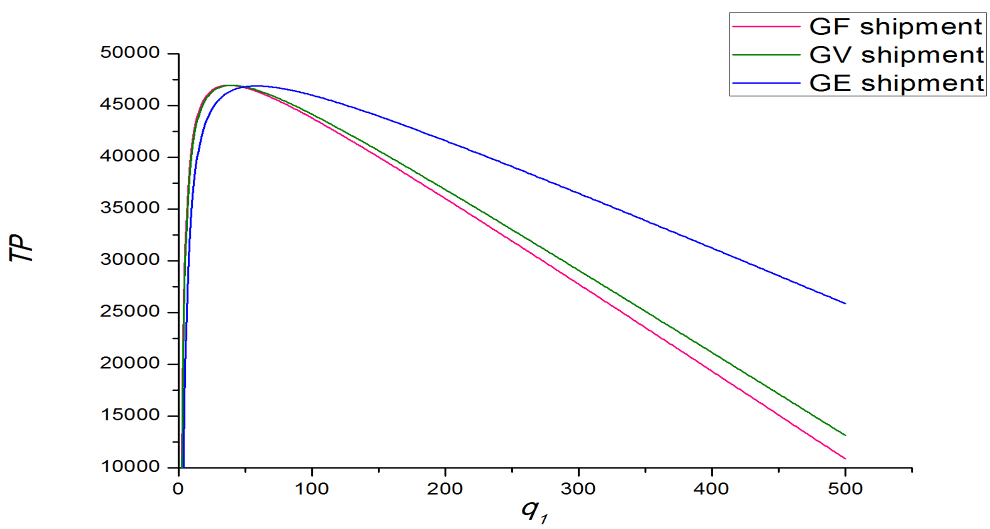

Figure 3.7: Plot of TP^* against q_1 when $n_v = 3$, $n_b = 2$, $n_r = 2$, and $\lambda = 2.2675/\frac{P}{\alpha}$.

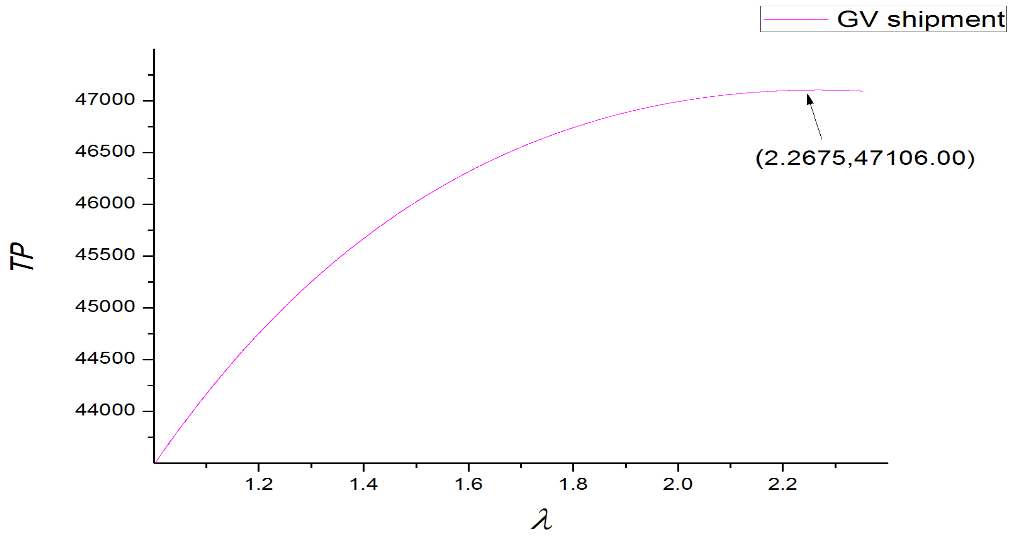

Figure 3.8: Plot of TP^* against λ when $n_v = 3$, $n_b = 1$, $n_r = 2$, and $q_1 = 71.988$.

The results were obtained by adding the equal shipment policy together with the proposed models to show the improvement of the optimal results obtained. The numerical data is given in the Table 3.1. We use $\beta \in [0.0, 0.01, ..., 0.1]$ to analyze the effect of stock dependent demand as we discover the demand becomes more sensitive when β is higher than 0.1. The results are presented in Tables 3.2 to 3.4. The optimal result for GF policy is given in parentheses.

Table 3.1: Input Parameters values

P	4000(units/year)	h_d	17 (\$/unit/year)	h_r	7 (\$/unit/year)
A_v	400 (\$/setup)	h_v	9 (\$/unit/year)	A_r	100 (\$/installment)
A_b	100(\$/shipment)	h_w	11(\$/unit/year)		
S	25(\$/transfer)	σ	30(\$/unit)		
α	1700(units)	C_d	500(units)		

The results show that as β increases, the total maximum profit, TP^* for all policies increase. This is the common behavior since we already expect the stock level is proportional to the total profit. In Table 3.2, the transfer quantity cannot increase further for values of β above 0.06 because of the limited capacity of the display area. From the result in Table 3.4, we conclude that the total profit of GV policy is superior to GF policy. However, when $\beta = 0.02$ to 0.1, the TP^* for both policies are equal as λ become more sensitive to the stock-level.

Table 3.2: Computational results for (ES) policy

β	TP	q_1	n_b	n_v	n_r	$\sum_{i=1}^{n_v} Q_1$
0	44767.90	95.47	2	3	2	572.84
0.01	46797.90	194.69	1	3	2	584.07
0.02	49041.60	272.28	1	2	2	544.55
0.03	51555.70	302.58	1	2	2	605.17
0.04	54266.50	337.52	1	2	2	675.05
0.05	57194.70	377.71	1	2	2	755.42
0.06	60395.40	453.49	1	2	3	906.98
0.07	63900.40	500	1	2	3	1000
0.08	67623.70	500	1	2	3	1000
0.09	71532.80	500	1	2	3	1000
0.1	75636.60	500	1	2	3	1000

Table 3.3: Computational results for (GE) policy

β	TP	q_1	n_b	n_v	n_r	$\sum_{i=1}^{n_v} Q_1$	$TP_{GE} - TP_{ES}$	PG
0	45067.80	52.735	2	3	2	738.296	299.90	0.670
0.01	47118.80	107.336	1	3	2	751.352	320.90	0.686
0.02	49434.90	120.344	1	3	2	842.408	393.30	0.802
0.03	52055.70	191.082	1	2	2	764.328	500	0.970
0.04	54902.50	215.545	1	2	2	862.180	636.00	1.172
0.05	58005.90	262.406	1	2	3	1049.624	811.20	1.418
0.06	61442.90	299.091	1	2	3	1196.364	1047.50	1.734
0.07	65201.30	359.820	1	2	4	1439.280	1300.90	2.036
0.08	69381.70	414.012	1	2	4	1656.048	1758.00	2.600
0.09	73982.60	478.588	1	2	5	1914.352	2449.80	3.425
0.1	79044.40	500	1	2	5	2000	3407.80	4.505

Table 3.4: Computational results for $(GF$ and $GV)$ policies

β	TP	q_1	n_b	n_v	n_r	$\sum_{i=1}^{n_v} Q_1$	$TP_{GS} - TP_{ES}$	PG	λ
0	45062.40	35.042	2	3	2	601.246	294.50	0.6578	2.2980
	(45062.20)	(33.818)	(2)	(3)	(2)	(601.230)	(294.30)	(0.6574)	(2.3529)
0.01	47106.00	71.988	1	3	2	605.357	308.10	0.6584	2.2675
	(47105.60)	(68.020)	(1)	(3)	(2)	(604.645)	(307.70)	(0.6575)	(2.3529)
0.02	49503.50	76.421	1	3	2	679.324	461.90	0.9419	2.3529
0.03	52098.60	86.459	1	3	2	768.556	542.90	1.0530	2.3529
0.04	54962.60	106.144	1	3	3	943.543	696.10	1.2827	2.3529
0.05	58108.50	122.112	1	3	3	1085.487	913.80	1.5977	2.3529
0.06	61595.70	149.374	1	3	4	1327.826	1200.30	1.9874	2.3529
0.07	65477.00	182.278	1	3	5	1620.319	1576.60	2.4673	2.3529
0.08	69837.20	223.220	1	3	6	1984.264	2213.50	3.2733	2.3529
0.09	74985.70	138.629	1	4	9	3038.184	3452.90	4.8270	2.3529
0.1	81113.90	89.044	1	5	14	4680.779	5477.30	7.2416	2.3529

To represent gains from using different shipment policies versus equal shipment policy, we define the percentage gain as $PG = \frac{100(TP_{GE/GV/GF} - TP_{ES})}{TP_{ES}}$. The results also show that the different shipment sizes policies are superior compare to equal shipment sizes policy .This statement can be shown by looking at the percentage gain, PG obtained in Table 3.3 and 3.4. For example, when $\beta = 0.01$, the PG of the GE is 0.686 , 0.6584 for GV, and 0.6575 for GF policy.

The comparison results in Tables 3.3 and 3.4 show that TP^* of GE policy is better than TP^* of GV and GF policies until $\beta = 0.01$. When $\beta = 0.02$ to 0.1, the result obtained is vice-versa. Another conclusion that can be summarized from Table 3.2 to 3.4 is the transfer quantity and total production is proportional to the β except for both (GV) and (GF) policies, the transfer quantity start to decline at the value of $\beta = 0.08$ due to the increment in n_v. Nevertheless, the different outcomes may be obtained while carrying different parameters values.

Example 3.4.2: The total profit is a solution of the model where the model parameters (holding cost, setup cost, production, and demand rates) are assumed to be fixed. By carrying the sensitivity analysis for the parameters, the effect of the changes of system parameters values on the total profit can be studied by increasing these parameter values but keep the other current parameter values the same except for $\alpha = 1800$, $P = 4500$ and $\beta = 0.05$. We only consider policies for geometry equal shipment, GE and geometry with variable λ, GV as the results for geometry with fixed λ, GF is almost the same as GV.

The sensitivity analysis for parameters A_b, A_v, S, A_r, h_d, h_v, and h_r are summarized in Figures 3.9 until 3.16 from the results in Tables 3.5 until 3.12. From the figures, by increasing the values of A_b, A_v, S, A_r, h_d, h_v, and h_r, the optimum total profit for all policies decrease. All of these results discussed are based on a specific parameter values. Nevertheless, the different outcomes may be obtained while carrying different parameters values. For example, the total profit for geometric then equal shipment (GE) could be better than geometric with variable λ shipment (GV).

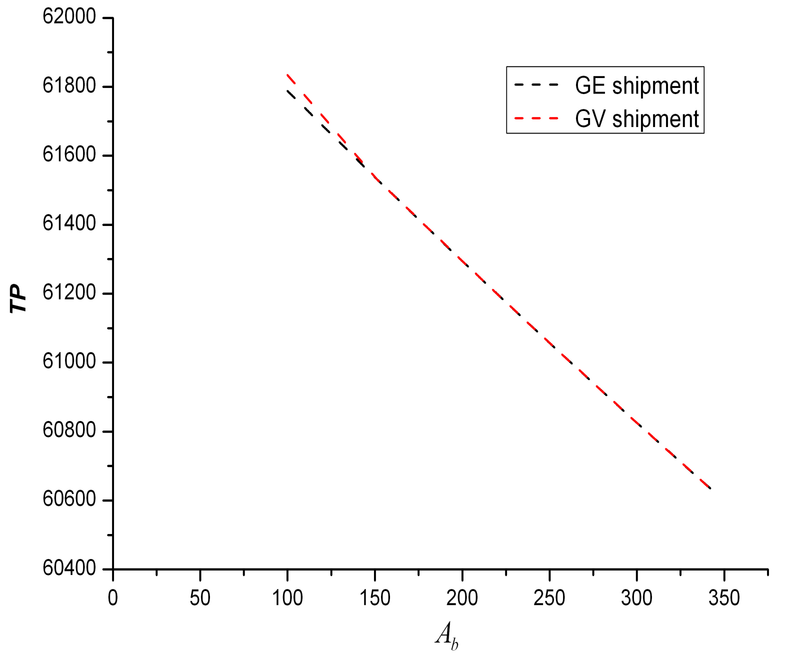

Figure 3.9: Plot of TP^* against A_b.

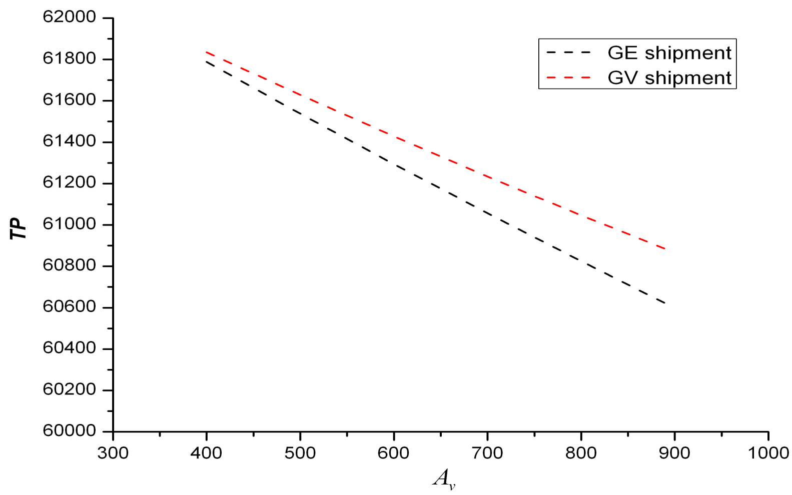

Figure 3.10: Plot of TP^* against A_v.

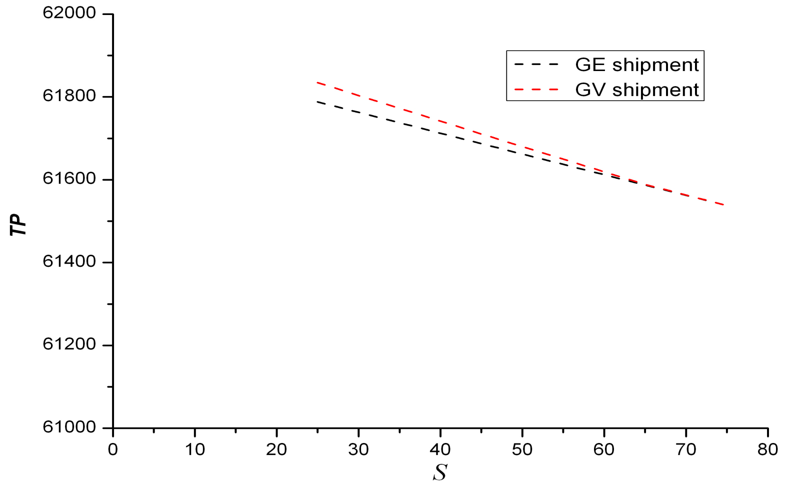

Figure 3.11: Plot of TP^* against S.

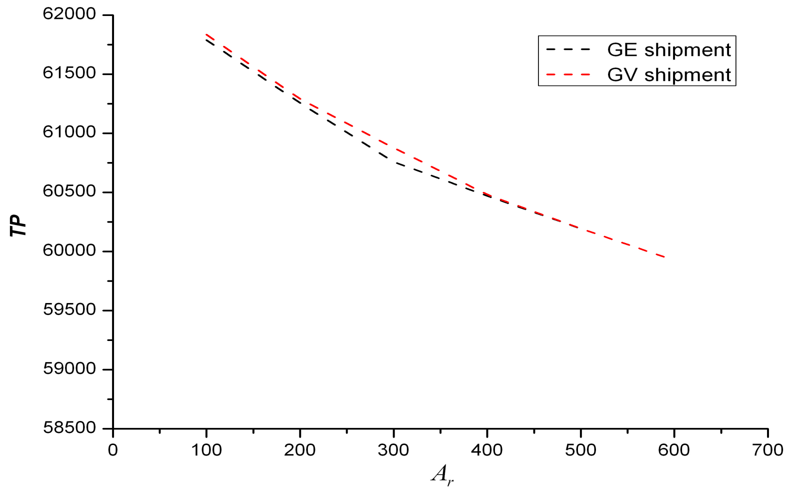

Figure 3.12: Plot of TP^* against A_r.

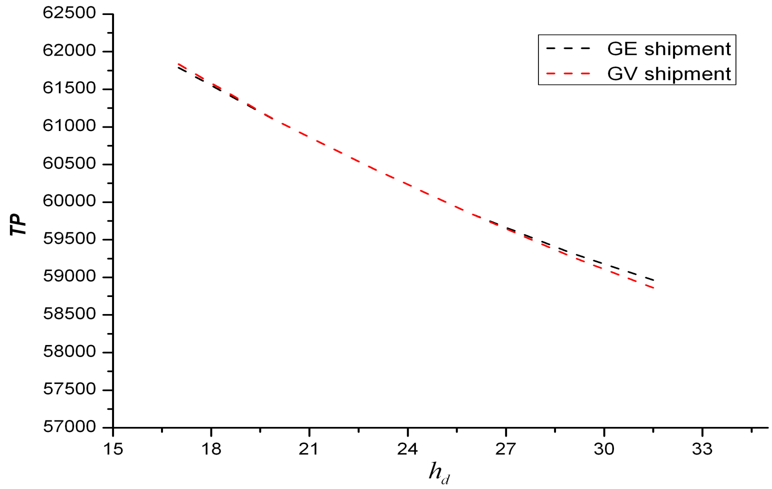

Figure 3.13: Plot of TP^* against h_d.

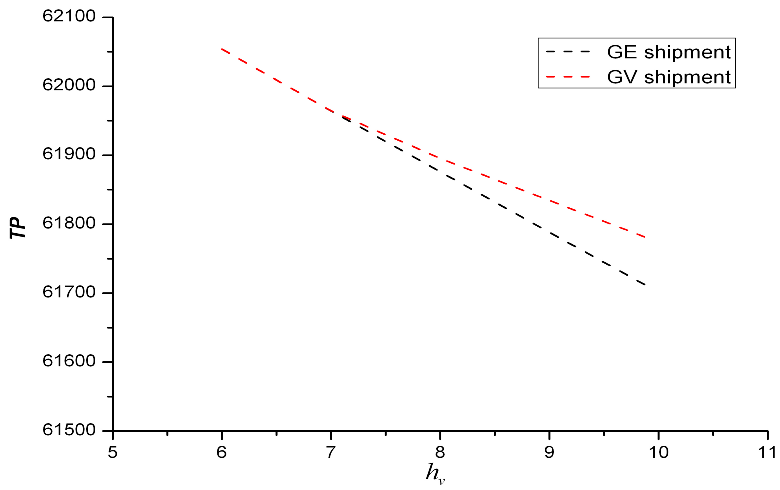

Figure 3.14: Plot of TP^* against h_v.

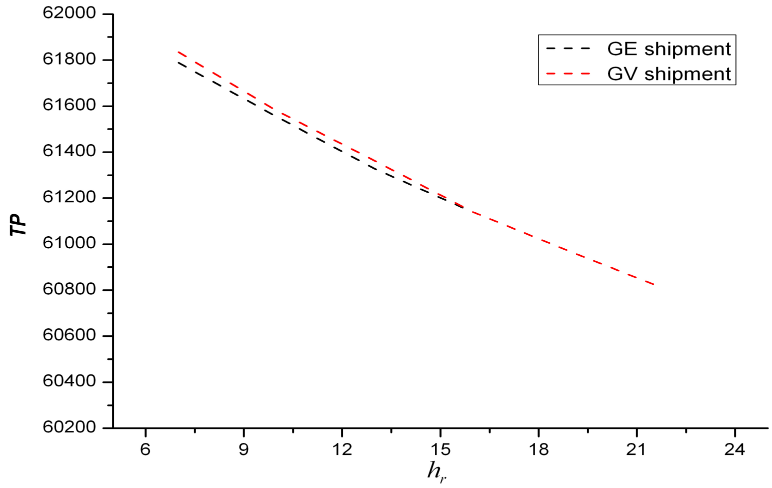

Figure 3.15: Plot of TP^* against h_r.

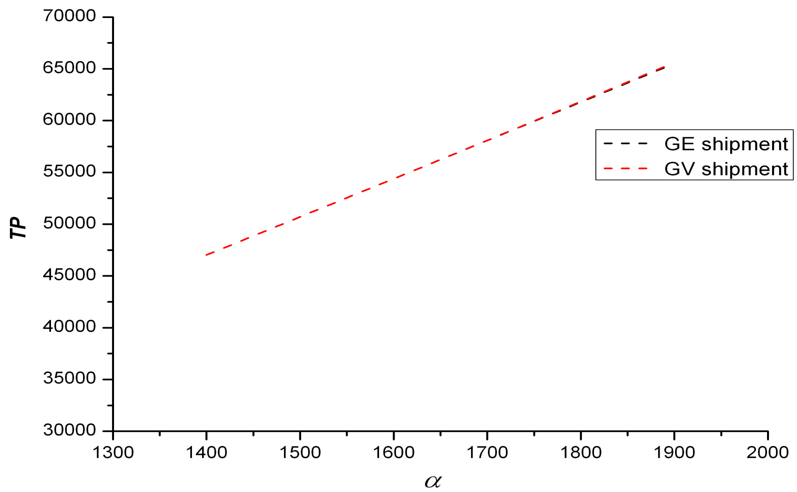

Figure 3.16: Plot of TP^* against α.

Table 3.5: Optimal solutions for varying A_b (GE and GV policies).

	GE					GV					
A_b	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
100	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
150	61537.70	270.400	1	2	3	61537.70	270.400	1	2	3	2.5
200	61294.10	278.071	1	2	3	61294.10	278.071	1	2	3	2.5
250	61056.60	285.468	1	2	3	61056.60	285.468	1	2	3	2.5
300	60824.80	292.618	1	2	3	60824.80	292.618	1	2	3	2.5
350	60598.20	299.547	1	2	3	60598.20	299.547	1	2	3	2.5

Table 3.6: Optimal solutions for varying S (GE and GV policies).

	GE					GV					
S	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
25	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
35	61737.50	264.045	1	2	3	61772.30	115.714	1	3	3	2.5
45	61687.20	265.653	1	2	3	61710.70	116.621	1	3	3	2.5
55	61637.10	267.248	1	2	3	61649.50	117.518	1	3	3	2.5
65	61587.30	268.830	1	2	3	61588.80	118.406	1	3	3	2.5
75	61537.70	270.400	1	2	3	61537.70	270.400	1	2	3	2.5

Table 3.7: Optimal solutions for varying A_v (GE and GV policies).

	GE					GV					
A_v	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
400	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
500	61537.70	270.400	1	2	3	61629.20	117.815	1	3	3	2.5
600	61294.10	278.071	1	2	3	61428.90	120.729	1	3	3	2.5
700	61056.60	285.468	1	2	3	61233.10	123.552	1	3	3	2.5
800	60824.80	292.618	1	2	3	61046.50	132.022	1	3	3	2.5
900	60598.20	299.547	1	2	3	60866.40	134.656	1	3	3	2.5

Table 3.8: Optimal solutions for varying A_r (GE and GV policies).

	GE					GV					
A_r	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
100	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
200	61256.40	260.835	1	2	2	61290.90	112.541	1	3	2	2.5
300	60758.70	275.551	1	2	2	60877.30	118.111	1	3	2	2.5
400	60469.80	236.788	1	2	1	60481.30	123.350	1	3	2	2.5
500	60193.60	243.816	1	2	1	60193.60	243.816	1	2	1	2.5
600	59924.60	250.586	1	2	1	59924.60	250.586	1	2	1	2.5

Table 3.9: Optimal solutions for varying h_d (GE and GV policies).

	GE					GV					
h_d	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
17	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
20	61079.50	223.695	1	2	2	61079.50	223.695	1	2	2	2.5
23	60433.50	206.599	1	2	2	60433.50	206.599	1	2	2	2.5
26	59834.10	192.487	1	2	2	59834.10	192.487	1	2	2	2.5
29	59322.00	134.677	1	3	2	59273.70	180.615	1	2	2	2.5
32	58892.20	127.963	1	3	2	58777.30	117.337	1	3	2	1.86807

Table 3.10: Optimal solutions for varying h_v (GE and GV policies).

	GE					GV					
h_v	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
6	62053.90	271.063	1	2	3	62053.90	271.063	1	2	3	2.5
7	61964.40	268.108	1	2	3	61964.40	268.108	1	2	3	2.5
8	61875.90	265.229	1	2	3	61895.30	115.571	1	3	3	2.5
9	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
10	61701.40	259.687	1	2	3	61773.80	114.037	1	3	3	2.5

Table 3.11: Optimal solutions for varying h_r (GE and GV policies).

	GE					GV					
h_r	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
7	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
10	61554.00	254.327	1	2	3	61579.90	117.022	1	3	4	2.5
13	61327.10	246.857	1	2	3	61362.10	113.618	1	3	4	2.5
16	61139.00	257.003	1	2	4	61139.00	257.003	1	2	4	2.5
19	60966.20	251.524	1	2	4	60966.20	251.524	1	2	4	2.5
22	60797.20	246.346	1	2	4	60797.20	246.346	1	2	4	2.5

Table 3.12: Optimal solutions for varying α(GE and GV policies).

	GE					GV					
α	TP	q_1	n_b	n_v	n_r	TP	q_1	n_b	n_v	n_r	λ
1400	47019.80	169.178	1	2	2	47019.80	169.178	1	2	2	3.214
1500	50698.80	187.638	1	2	2	50698.80	187.637	1	2	2	3.0
1600	54386.50	206.462	1	2	2	54386.50	206.462	1	2	2	2.813
1700	58081.50	225.575	1	2	2	58081.50	225.575	1	2	2	2.647
1800	61788.20	262.423	1	2	3	61834.40	114.798	1	3	3	2.5
1900	65516.80	283.783	1	2	3	65606.90	131.427	1	3	3	2.368

Chapter 4

Single-Supplier, Single-Vendor and multiple-buyer

In this chapter, we extend our previous model by considering a single-suppier, single-vendor and multiple-buyer. For this model, non-coordinated and coordinated cases are considered and both performances are compared by using Premium Solver in Microsoft Excel.

4.1 Assumptions

In this model, we only considering equal shipment sizes policy. Most of the assumptions for the first model are retained, except now we have multiple-buyer. For more convenience, we reinstated again.

1. The demand rate $D_k(t)$ is assumed to be in the form as in Baker and Urban (1988). For a buyer k, the functional relationship is given by

$$D_k(t) = \alpha_k [I_k(t)]^{\beta},$$

   where $\alpha_k > 0$ is the scale parameter, $I_k(t)$ is the inventory level at time t and $\beta \in (0, 1)$ is the shape parameter and is the measure of responsiveness of the demand rate to change in the inventory level. We note here, the demand rate is equal for every period for each buyer.

2. Shortages at the buyers' warehouse and display area are not allowed.

3. Time horizon is infinite.

4. There is a limited capacity $C_{d,k}$ of the display area, i.e $I \leq C_{d,k}$. This limitation could be interpreted as a given shelf space, allocated to the product.

5. The vendor has a finite production rate P which is greater than the maximum possible demand rate, i.e $P > M$ where $M = \sum_{k=1}^{Y}(\alpha_k C_{d,k})^{\beta}$.

6. For simplicity we only consider one type of raw material is required to produce one unit of a finished product.

4.2 Notations

We adopted the similar notations except the notations for the buyers where

1. A_v Setup cost per production for vendor.

2. $A_{b,k}$ kth buyer fixed shipment cost.

3. A_r Fixed installment cost for raw material.

4. S_k kth buyer fixed transferring cost from the warehouse to the display area.

5. c The net unit purchasing price (charged by the vendor to the buyers).

6. σ_k kth buyer net unit selling price (charged by the buyer to the customers).

7. h_r The raw material holding cost per unit time.

8. h_v The inventory holding cost per unit time at the vendor.

9. $h_{w,k}$ kth buyer inventory holding cost per unit time at the buyer's warehouse where $h_{w,k} > h_v$.

10. $h_{d,k}$ kth buyer inventory holding cost per unit time at the buyer's display area
where $h_{d,k} > h_{w,k}$.

11. n_r The number of installment.

12. $n_{v,k}$ kth buyer number of shipment.

13. $n_{b,k}$ kth buyer number of transfer.

14. Q_k kth buyer shipment lot size from vendor to buyer warehouse.

15. q_k kth buyer transfer lot size from warehouse to display area.

16. Y Number of buyers, where $k = 1, 2, ..., Y$.

17. T^* Total cycle time at the vendor's inventory.

18. T_k Total cycle time at kth buyer's inventory.

4.3 General formulation

The inventory level time plot for this model is depicted in Figure 4.1. The top part of the figure shows the inventory level of raw materials delivered in $n_r = 3$ equal lot installment during production uptime. The bottom part of the figure shows the inventory level of the supplier, vendor and three-buyers with $n_{v,1} = 3, n_{b,1} = 3, n_{v,2} = 1, n_{b,2} = 4, n_{v,3} = 2$, and $n_{b,3} = 4$.

We let $I_k(t)$ be the display area inventory level at time t, and we have

$$\frac{dI_k(t)}{dt} = -\alpha_k [I_k(t)]^{\beta}, \qquad 0 \le t \le T_{d,k}, k = 1, 2, ..., Y, \tag{4.1}$$

where $T_{d,k}$ is the period time defined in Figure 4.1 with $I_k(0) = q_k$ and $I_k(T_{d,k}) = 0$.

Solving differential equation (4.1), we get

$$I_k(t) = \Big[- \alpha_k(1-\beta)t + q_k^{1-\beta} \Big]^{\frac{1}{1-\beta}}.$$

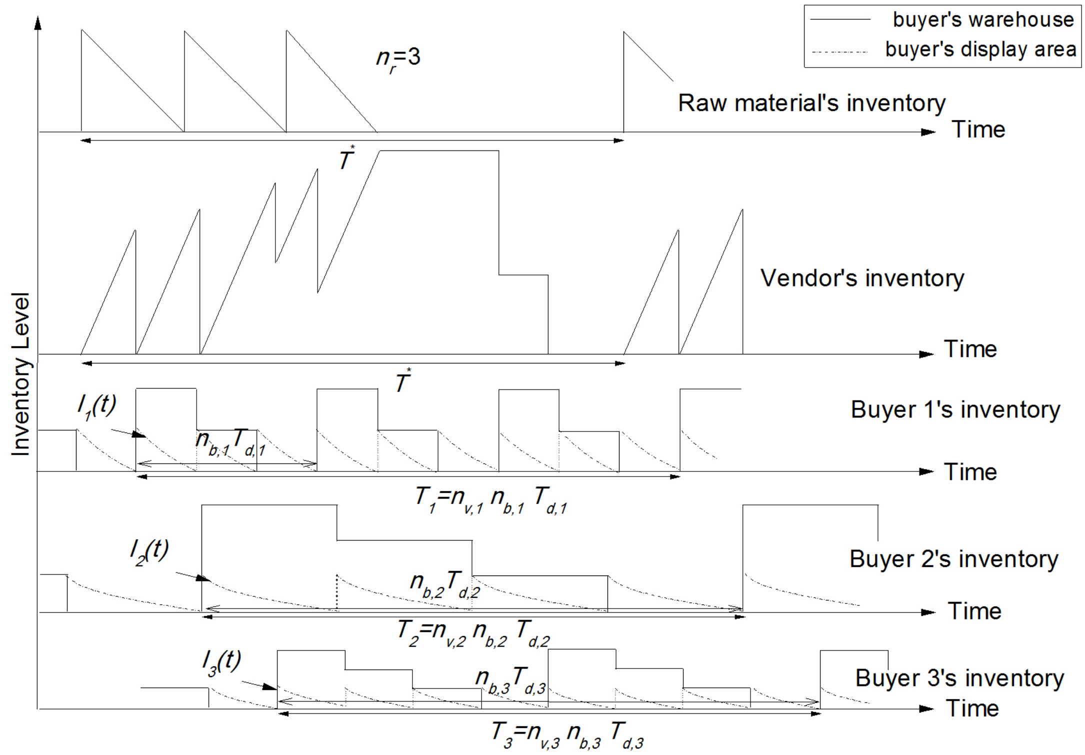

Figure 4.1: A graphical representation of integrated supply chain with single-vendor multiple-buyer.

4.3.1 Non-coordinated supply chain

This section deals with the derivation of independently supply chain for the vendor and the buyers. The objective is to maximize their total profit. Then, the results will be compared with the results from coordination supply chain.

4.3.1.1 Buyer's average total profit formulation

At first shipment, buyer k will receive Q_k amount of an item, keep the items inside the warehouse and transfer $n_{b,k}$ times q_k of amount until the items at the warehouse reach zero. The time taken to consume Q_k is $n_{b,k}T_{d_k}$. Thus, buyer k's inventory at the warehouse for the first shipment is $[(n_{b,k}-1)Q_kT_{d_k}]$. This continues until the entire shipment is supplied. We assume that there are $n_{v,k}$ shipment in a lot, thus the total average inventory at the warehouse per cycle is given by

$$I_{bw,k} = \frac{1}{2T_k}\Big[(n_{b,k}-1)n_{v,k}Q_kT_{d_k}\Big] \tag{4.2}$$

where T_k is the total cycle time of the supply chain and from Figure 4.1, we have

$$T_k = n_{b,k}n_{v,k}T_{d,k}$$

and

$$T_{d,k} = q_k^{1-\beta}/[\alpha_k(1-\beta)]$$

By solving and simplify Equation(4.2), the average inventory at the warehouse can be expressed as follows:

$$I_{bw,k} = \frac{(n_{b,k}-1)q_k}{2} \tag{4.3}$$

The total holding cost at the warehouse is given by

$$\begin{aligned} HC_{bw,k} &= h_wI_{bw} \\ &= \frac{h_w(n_{b,k}-1)q_k}{2}. \end{aligned} \tag{4.4}$$

Now, the expression of the average inventory at the display area needs to be defined. From Figure 4.1, it can be observed that the inventory holding of the items is given by $n_{b,k}n_{v,k}\int_0^{T_{d,k}} I_k(t)dt$. Then, it follows that the average inventory at the display area for buyer k, $I_{bd,k}$ is given by

$$I_{bd,k} = \frac{1}{T_k} n_{b,k}n_{v,k}\int_0^{T_{d,k}} I_k(t)dt. \tag{4.5}$$

Then, by solving and simplifying the expression above, the equation of the average inventory at the display area can be rewritten as

$$I_{bd,k} = \frac{(1-\beta)q_k}{2-\beta}. \tag{4.6}$$

The total holding cost at the display area is given by

$$\begin{aligned} HC_{bd,k} &= h_d I_{bd,k} \\ &= \frac{h_d(1-\beta)q_k}{2-\beta} \end{aligned} \tag{4.7}$$

Finally, the total relevant cost for buyer k,

$$TRC_{b,k} = \frac{1}{T_k}(n_{v,k}A_{b,k} + n_{v,k}n_{b,k}S_k) + HC_{bw,k} + HC_{bd,k}. \tag{4.8}$$

where the first term represents average shipment cost and average transfer cost.

For each order with a quantity of Q_k, buyer k is charged cQ_k from the vendor, and receives the amount $\gamma_k Q_k$ from the customer. Therefore the buyer's total sale revenue per unit time is $TR_b = \frac{(\gamma_k - c)n_{v,k}Q_k}{T_k}$ and the buyer's total profit is $TP_{b,k} = TR_{b,k} - TRC_{b,k}$. The total profit for kth buyer is a function of $n_{b,k}$, $n_{v,k}$, and q_k. Thus, for Y buyers,

$$\begin{aligned} TP_{b,k}(n_{b,k}, n_{v,k}, q_k) &= \sum_{k=1}^{Y}\frac{(\gamma_k - c)n_{v,k}n_{b,k}q_k}{T_k} - \Big[\sum_{k=1}^{Y}\frac{1}{T_k}(n_{v,k}A_{b,k} \\ &+ n_{v,k}n_{b,k}S_k) + \sum_{k=1}^{Y}\frac{h_w(n_{b,k}-1)q_k}{2} \end{aligned}$$

$$+ \quad \sum_{k=1}^{Y} \frac{h_d(1-\beta)q_k}{2-\beta}\Big]. \tag{4.9}$$

4.3.1.2 Vendor's average total profit formulation

At the end of a production run, all units of raw material will be fully consumed. Let $\acute{\psi}$ be the summation of total production quantity for buyer k, and from Figure 4.1, the maximum inventory of raw material for each installment is $\acute{\psi}/n_r$ and lasting for the period of $\acute{\psi}/(n_r P)$ units time. It follows that the average inventory of raw material, $I_{r,k}$ is

$$I_{r,k} = \frac{1}{T^*}(\acute{\psi}/n_r)(\acute{\psi}/2n_r P)n_{r,k} = \frac{1}{2n_r PT^*}(\acute{\psi})^2. \tag{4.10}$$

where $\acute{\psi} = \sum_{k=1}^{Y} \tilde{\psi}_k$ and $\tilde{\psi}_k = n_{v,k}Q_k$. Simplified, then

$$I_{r,k} = \frac{1}{2n_r PT^*}\Big(\sum_{k=1}^{Y}(n_{v,k}Q_k)^2\Big). \tag{4.11}$$

Thus, the holding cost of raw material is

$$\begin{aligned} HC_r &= h_r I_{r,k} \\ &= \frac{h_r}{2n_r PT^*}\Big(\sum_{k=1}^{Y}(n_{v,k}Q_k)^2\Big). \end{aligned} \tag{4.12}$$

Now, by following the same method as in Hill and Omar (2006), the average finished product at the vendor, I_v is $\frac{\tilde{\psi}_k}{2} - \frac{\tilde{\psi}_k^2}{2T_k P} + \frac{\tilde{\psi}_k Q_{1,k}}{T_k P} - \frac{n_{b,k}}{2T_k} n_{v,k}Q_k T_{d,k}$ where the first three terms represent the average system stock and the final term represent the average stock with the shipment size of $n_{v,k}Q_k$. Thus, by substitution, we get

$$I_{v,k} \quad = \quad \frac{(n_{v,k}-1)n_{b,k}q_k}{2} - \frac{(n_{v,k}n_{b,k}q_k)^2}{2T_k P} + \frac{n_{v,k}(n_{b,k}q_k)^2}{T_k P}. \tag{4.13}$$

It follows that the total holding cost at the vendor is

$$
\begin{aligned}
HC_v &= h_v I_{v,k} \\
&= h_v\Big[\frac{(n_{v,k}-1)n_{b,k}q_k}{2} - \frac{(n_{v,k}n_{b,k}q_k)^2}{2T_kP} + \frac{n_{v,k}(n_{b,k}q_k)^2}{T_kP}\Big]. \qquad (4.14)
\end{aligned}
$$

Finally, the total relevant cost at the vendor,

$$TRC_v = \frac{1}{T^*}(n_rA_r + A_v) + HC_r + HC_v. \qquad (4.15)$$

where the first term represents average setup cost and average installment cost for the vendor.

In this supply chain system, we assume that the selling price for all buyers are the same. The vendor produces $n_{v,k}Q_k$ products and sells to buyer k at price c per unit product. Therefore, the vendor's sale revenue per unit time is $TR_v = \frac{cn_{v,k}Q_k}{T^*}$ and total profit for vendor is $TP_v = TR_v - TRC_v$. Thus, for Y buyers,

$$
\begin{aligned}
TP_v(n_r) &= \sum_{k=1}^{Y}\frac{cn_{v,k}n_{b,k}q_k}{T^*} - \Big[\frac{1}{T^*}(n_rA_r + A_v) + \frac{h_r}{2n_rPT^*}\Big(\sum_{k=1}^{Y}(n_{v,k}n_{b,k}q_k)^2\Big) \\
&+ h_v\sum_{k=1}^{Y}\Big(\frac{(n_{v,k}-1)n_{b,k}q_k}{2} - \frac{(n_{v,k}n_{b,k}q_k)^2}{2T_kP} \\
&+ \frac{n_{v,k}(n_{b,k}q_k)^2}{T_kP}\Big)\Big]. \qquad (4.16)
\end{aligned}
$$

Then, the average system total profit for non-coordinated supply chain, TP_n can be obtained as $TP_n = TP_b(q_k^*, n_{b,k}^*, n_{v,k}^*) + TP_v(n_r^*)$.

4.3.2 Coordinated supply chain

Once the vendor and buyer have established long-term strategic partnership and contracted to commit the relationship, they will jointly determine the best policies for the whole supply chain system. Therefore, the total joint profit per unit time,

TP for the integrated model is

$$TP_c = TP_v + TP_{b,k}. \tag{4.17}$$

By substituting Equations (4.9), and (4.16) into (4.17) and simplifying the equation, we get

$$\begin{aligned} TP_c &= \sum_{k=1}^{Y} \frac{\gamma_k n_{v,k} n_{b,k} q_k}{T_k} - \Big[\sum_{k=1}^{Y} \frac{1}{T_k} (n_{v,k} A_{b,k} + n_{v,k} n_{b,k} S_k) \\ &+ \sum_{k=1}^{Y} h_{w,k} \frac{(n_{b,k}-1) q_k}{2} + \sum_{k=1}^{Y} h_{d,k} \frac{(1-\beta) q_k}{2-\beta} \Big] \\ &- \frac{1}{T^*}(n_r A_r + A_v) - \frac{h_r}{2 n_r P T^*} \sum_{k=1}^{Y} (n_{v,k} n_{b,k} q_k)^2 \\ &- h_v \sum_{k=1}^{Y} \Big(\frac{(n_{v,k}-1) n_{b,k} q_k}{2} - \frac{(n_{v,k} n_{b,k} q_k)^2}{2 T_k P} \\ &+ \frac{n_{v,k} (n_{b,k} q_k)^2}{T_k P} \Big). \end{aligned} \tag{4.18}$$

The order cycle lengths for the buyers commonly are different, i.e., $T_1 \neq T_2 \neq ... \neq T_Y$. In order to coordinate the inventory systems in a supply chain, the vendor must convince the buyers to order duly to the fixed schedule, i.e., $T^* = T_k$ where $k = 1, 2, ..., Y$. Thus, such a policy will alleviate the amalgamation of the buyers' orders by the vendor.

4.4 Solution procedures

In this section, we obtain the non-coordinated optimal solutions for the vendor and the buyer, and also for the coordinated optimal solutions, respectively. We employ a one-dimensional search algorithm to find the optimal values of the decision variables by using Microsoft Excel's Premium Solver tool to solve the maximization problem. Solver analyses the problem as a nonlinear one and uses "Standard LSGRG Nonlinear" engine to solve the problem.

4.4.1 Solution procedure for non-coordinated problem

The step to find the optimal values of total profit at the buyers is obtained below:

Step 1: Initialize by putting $n_{b,k} = 1$, $n_{v,k} = 1$, and $q_k = 1$.

Step 2: Set TP_b as the target cell equal to max by changing parameter $n_{b,k}$, $n_{v,k}$, and q_k subject to the constraints;

i. $q_k \geq 1$ & $q_1 \leq C_{d,k}$.

ii. $n_v \geq 1$ & $n_b \geq 1$.

iii. $\{n_{v,k}, n_{b,k},\} \in Integers$.

iv. $T_1 = T_2 = ... = T_k$

The kth buyer will find the optimal policy of q_k^*, $n_{b,k}^*$, and $n_{v,k}^*$. Then, the vendor will choose the optimal number of installment n_r^* by putting value of $T = T_k$. Therefore, the optimal system total profit per unit time is $TP_n = TP_b(q_k^*, n_{b,k}^*, n_{v,k}^*) + TP_v(n_r^*)$.

4.4.2 Solution procedure for coordinated problem

The computer algorithm of the solution procedure for coordinated supply chain is outlined below.

Step 1: Initialize by putting $n_{b,k} = 1$, $n_{v,k} = 1$, $q_k = 1$, $n_r = 1$, and $T = 1$.

Step 2: Set TP_c as the target cell equal to max by changing parameter $n_{b,k}$, $n_{v,k}$, q_k, n_r, and T subject to the constraints;

i. $q_k \geq 1$ & $q_1 \leq C_{d,k}$.

ii. $n_v \geq 1$ & $n_b \geq 1$.

iii. $\{n_{v,k}, n_{b,k},\} \in Integers$.

iv. $T = T_k$

Thus, the optimal policy for coordinated total profit is obtained as:

$$TP_c(q_k^*, n_{b,k}^*, n_{v,k}^*, T^*).$$

4.5 Numerical examples

A number of examples are made to test the solution of the model.

Example 4.5.1: Consider a three-level supply chain with four buyers ($k = 1, 2, 3, 4$), a vendor (manufacturer), and a supplier. In order to analyze the effect of stock dependent demand, we used $\beta \in [0.00, 0.05, ..., 0.2]$. We assume a system with parameter values of $P = 4500$, $A_v = 400$, $A_r = 200$, $h_v = 4$, and $h_r = 12$. For the buyers, the input parameters are given in Table 4.1. The results are presented in Tables 4.2 and 4.3.

Table 4.1: Input Parameter values (Single-supplier,single-vendor, and multiple-buyer)

k	$A_{b,k}$	S_k	$h_{w,k}$	$h_{d,k}$	α_k	σ_k	$C_{d,k}$
1	100	25	8	20	100	30	500
2	150	30	10	18	150	20	400
3	120	20	9	15	180	28	300
4	200	35	11	22	114	35	600

To represent gains from using non-coordinated supply chain versus coordinated supply chain, we define the percentage gain as $PG = 100(TP_c - TP_n)/TP_n$. Tables 4.2 and 4.3 show the values of total profit for different β. As expected, when β increases, TP_n and TP_c also increase. The numerical results also show that the total production, $\acute{\psi}$ is proportional to the increment of β. Other than that, the total lot size of q_k for each buyer in both cases increase as the β increases.

The comparison result between Tables 4.2 and 4.3 shows that TP_c of coordinated supply chain is better than TP_n of non-coordinated supply chain due to the positive value of percentage gain, PG obtained. Another conclusion that can be summarized is the optimal total ordering quantity for the buyers, $\acute{\psi}$ is always superior in the non-coordinated problem compared to the coordinated problem.

Table 4.2: Computational results for non-coordinated problem

β	k	q_k	$n_{v,k}$	$n_{b,k}$	$n_{r,k}$	TP_b	TP_v	TP_n	T^*	$\tilde{\psi}_k$	$\acute{\psi}$
0	1	17.332	2	3	2	6147.39	3842.42	9989.81	1.040	103.991	565.710
	2	38.997	2	2						155.986	
	3	31.197	3	2						187.183	
	4	29.637	2	2						118.549	
0.05	1	29.119	2	2	2	7300.34	4500.39	11800.72	1.036	116.474	648.195
	2	44.620	2	2						178.480	
	3	73.180	3	1						219.540	
	4	33.425	2	2						133.700	
0.1	1	57.254	2	1	2	8999.41	5700.84	14700.25	0.849	114.507	646.660
	2	89.838	2	1						179.676	
	3	110.012	2	1						220.024	
	4	66.226	2	1						132.453	
0.15	1	87.027	2	1	3	11414.87	6977.31	18392.18	1.048	114.507	736.498
	2	87.027	3	1						269.514	
	3	173.770	2	1						220.024	
	4	101.532	2	1						132.453	
0.2	1	92.332	2	1	3	14500.36	9226.83	23727.20	0.934	184.664	1105.357
	2	92.332	3	1						276.996	
	3	192.506	2	1						385.011	
	4	258.685	1	1						258.685	

Table 4.3: Computational results for coordinated problem

β	k	q_k	$n_{v,k}$	$n_{b,k}$	$n_{r,k}$	TP_c	T^*	$\tilde{\psi}_k$	$\acute{\psi}$	PG
0	1	22.750	1	3	1	10224.12	0.682	68.250	371.279	2.346
	2	34.125	1	3				102.375		
	3	30.712	2	2				122.850		
	4	25.935	1	3				77.805		
0.05	1	33.361	1	2	1	12126.48	0.589	66.723	374.033	2.760
	2	51.121	1	2				102.243		
	3	128.477	1	1				128.477		
	4	38.295	1	2				76.590		
0.1	1	76.464	1	1	1	15087.56	0.551	76.464	431.817	2.635
	2	119.981	1	1				119.981		
	3	146.924	1	1				146.924		
	4	88.447	1	1				88.447		
0.15	1	113.347	1	1	2	19253.58	0.656	113.347	654.541	4.683
	2	182.632	1	1				182.632		
	3	226.324	1	1				226.324		
	4	132.238	1	1				132.238		
0.2	1	143.890	1	1	2	25257.35	0.666	143.890	852.247	6.449
	2	239	1	1				238.861		
	3	300	1	1				300.000		
	4	169.497	1	1				169.497		

Example 4.5.2: We perform a sensitivity analysis by solving many sample problems in order to identify how the maximum total joint profits respond to parameter changes by using $\beta = 0.05$. The results obtained are illustrated in Tables 4.4 to 4.7.

From Tables 4.4 until 4.7, by increasing the values of A_r, A_v, h_r, and h_v, the optimal total profit for both non-coordinated and coordinated cases decrease. It can be concluded that the increment of parameters do not affect much in percentage gain, PG as all of the values of PG are too small.

Table 4.4: Optimal solution for varying A_v (Single-supplier,single-vendor, and multiple-buyer).

A_v	TP_c	TP_n	PG
400	12126.48	11800.72	2.760
450	12042.26	11752.46	2.466
500	11959.25	11704.19	2.179
550	11865.63	11655.92	1.799
600	11783.32	11607.65	1.513
650	11721.58	11559.38	1.403

Table 4.5: Optimal solution for varying A_r (Single-supplier,single-vendor, and multiple-buyer).

A_r	TP_c	TP_n	PG
100	12298.76	11993.80	2.543
150	12211.96	11897.26	2.645
200	12126.48	11800.72	2.760
250	11991.91	11704.19	2.458
300	11959.25	11626.85	2.859
350	11877.38	11578.58	2.581

Table 4.6: Optimal solution for varying h_v (Single-supplier,single-vendor, and multiple-buyer).

h_v	TP_c	TP_n	PG
2	12193.88	12157.97	0.295
3	12128.02	11979.34	1.241
4	12126.48	11800.72	2.760
5	12119.46	11622.10	4.279
6	12112.45	11443.48	5.846
7	12106.81	11264.86	7.474

Table 4.7: Optimal solution for varying h_r (Single-supplier,single-vendor, and multiple-buyer).

h_r	TP_c	TP_n	PG
10	12179.53	11845.79	2.817
11	12152.93	11823.26	2.788
12	12126.48	11800.72	2.760
13	12100.18	11778.19	2.734
14	12074.03	11755.66	2.708
15	12048.02	11733.12	2.684

Chapter 5

Conclusion and Suggestion for Future Research

5.1 Conclusion

In this research, the researcher reviewed the inventory model of integrated vendor-buyer problem with stock-dependent demand. Then, the researcher extended the aforementioned model considering different shipment policy. The researcher also extended the said model with single-supplier, single-vendor and multiple buyers. Finally, the researcher performed some sensitivity analysis to study the robustness of the optimal total profits, which is subject to parameters change. For example, in the first model, when the fixed shipment cost for the buyer (A_b) increases, the system favours smaller the number of shipments (n_v). Similarly, when A_r increases, the number of raw material instalments also decreases. The mathematical formulation for the model mentioned was defined by Wolfram Mathematica and Premium Solver in Excel. A numerical optimization was applied inside the programs since the maximization problems involved value of decision variables. Numerical examples were given for explanation of the application in the proposed solution procedures. The obtained numerical results defined how parameters values change affect the behavior of the decision variables. Then, these results showed the concavity behavior of the total profit functions with respect to their integer decision variables.

Although such integrated models are well-studied, this seems to be the first time models that involve the stock-dependent demand with different shipment policies and multiple buyers case using four stocking points of the supply chain are formulated and numerically verified.

5.2 Suggestion for future research

The work may be extended in several ways. The first model can be extended by considering multiple buyers case. The different shipment policy for the single-vendor multiple buyers model is not investigated. Thus it is possible to extend the research in this direction. Another possible extension is to consider multiple types of products rather than one in the system as in the real world practices, each echelon handles more than one product at a time.

There are several possible directions our model could take for future research. One immediate extension would be to investigate the effect when the inventory at the displayed area deteriorates with time. It might be interesting to consider an inventory system with stock-dependent selling rate demand. We also might consider unit cash discount and delay payment (see, for example Teng et al., 2011). Finally we may extend the proposed model to account for more than one vendors and buyers (see Glock, 2011, 2012b, 2012c).

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